Final

Friday, 2 May 2025
11:00 AM – 1:30 PM

There are 7 problems each worth 6 points. No notes, no collaboration. Please respect and uphold the integrity of the examination process. Sign the cover page to show agreement with these instructions.

Name: _____

Problem	Credit
1	
2	
3	
4	
5	
6	
7	
Total	

1. **Boolean Algebra:**

   Simplify the boolean expression,

   $$(a \vee b) \vee \big((a \vee b) \wedge (c \vee d)\big)$$

   Hint: This is an example of the absorption law.

2. **Boolean Algebra:**

   Write a boolean expression using $\wedge, \vee$ and $\sim$ that realizes the boolean function,

$$f(a,b,c) = \begin{cases} T & \text{if most inputs are } T, \\ F & \text{else} \end{cases}$$

3. **NP Problems**

   (a) Given n integers $a_1, a_2, \ldots, a_n$ show that the problem of whether there are any repeated values among them is in NP.

   (b) Is the problem in P?

   You need to justify your answers.

4. **Context Free Grammars:**

   Write a context free grammar for the language of all strings in $\{ a, b, c \}^*$ that are of the form,

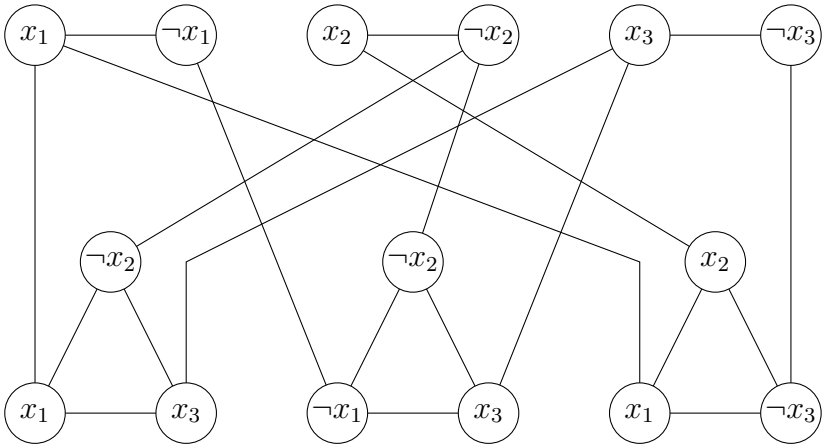
   $$a^i b^j c^k \text{ where } i = j + k$$

5. **NP Reductions:**

Given the 3-SAT formula,

$$(x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \neg x_3)$$

the reduction to k-VertexCover is given by this graph ($k = 9$),



Find a 9-cover (that is, 9 selected vertices) and also the associated satisfying assignment to the variables x_1, x_2 and x_3.

(a) Show the 9-cover by filling in the 9 vertices in the cover.

(b) The satisfying assignment,

- $x_1 =$ _____
- $x_2 =$ _____
- $x_3 =$ _____

6. **Decidable or Undecidable**

   Mark as Undecidable (U) or Decidable (D),

   (a) _____ Whether a string is in the set described by a Regular Expression (A_{FA}).

   (b) _____ Whether a string is in the set described by a Context Free Grammar (A_{CFG}).

   (c) _____ Whether a string is in the set described by a Turing Machine Program (A_{TM}).

   (d) _____ Whether two Regular Expressions give the same language (EQ_{FA}).

   (e) _____ Whether two Context Free Grammars give the same language (EQ_{CFG}).

   (f) _____ Whether two Turing Machine Programs give the same language (EQ_{TM}).

7. **RP algorithm**

   You are given two subroutines F and G which evaluate at the cost of one time step the polynomials $f(x)$ and $g(x)$, respectively.

$$\begin{aligned} F(x) &:= f(x) \\ G(x) &:= g(x) \end{aligned}$$

   You are also promised both polynomials are of degree at most d.

   We want an algorithm for the problem:

$$\textit{Is } F \neq G \textit{ ?}$$

   That is, is there an x such that $F(x) \neq G(x)$.

   Use the definition of RP to show that this problem is in RP.

   Hint: Use the degree d and explain.