Discrete Mathematics **Comprehensive Examination, Spring 2002**

Sign the exam with your student number - not your name _____

Answer all five questions to the best of your ability.

1. (20 pts) Given that the universe of discourse is the set of integers, determine the truth value of the following statements.

   1. $\forall n(n^2 \geq 0)$

   2. $\exists n \forall m(n < m^2)$

   3. $\exists n \forall m(nm = m)$

   4. $\exists n \exists m(n^2 + m^2 = 6)$

   5. $\forall n \forall m \exists p(p = (m + n)/2)$

2. (20 pts) Prove that
$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$$
whenever n is a positive integer greater than 1.

3. (20 pts) How many bit strings of length 8 have:

    1. exactly three 0's?

    2. the same number of 1's as 0's?

    3. an even number of 1's?

4. (20 pts) Using alphabetical order, construct a binary search tree for the words in the sentence:

   The quick brown fox jumped over the lazy dog.

   What is the maximum number of string compares needed to determine that a word is, or is not, in the sentence?

5. (20 pts) The cost to lease telephone lines between computers located in some cities is shown in the table below. Design a minimum-cost communication network so that there is some telecommunication path between all the computers.

From	To				
	Atlanta	Chicago	Denver	New York	San Francisco
Atlanta	-	$700	$1400	$800	$2200
Chicago	$700	-	$1300	$1000	$1200
Denver	$1400	$1300	-	$1600	$900
New York	$800	$1000	$1600	-	$2000
San Francisco	$2200	$1200	$900	$2000	-