Formal Languages
Comprehensive Exam
Fall 2012

1) (20 points) Select an answer of true (T) or false (F) for each of the following.

Let L be a language that is context-free but not regular. Then L is recursive.   T   F

Let M be a multi-tape, non-deterministic Turing machine. Then there exists a single-tape, deterministic Turing machine M' such that L(M) = L(M').

       T   F

Let M be a non-deterministic PDA. Then there exists a deterministic PDA M' such that L(M) = L(M').

       T   F

Let M be a Turing machine, and let w be a string such that M terminates on w. Because M terminates on w, we can therefore conclude that L(M) is recursive.

       T   F

The Church-Turing thesis states that every recursively-enumerable language is also recursive.   T   F

Let L be a language. The Church-Turing thesis states that there is a DFA M such that L = L(M).

       T   F

The Church-Turing thesis states that every language is context-free.   T   F

Let L be a language that is not context-free, not recursive, not context-sensitive and not recursively enumerable. Then L must be regular.

       T   F

Let L1 be a regular language, and let L2 be a recursive language. Then L1∩L2 is a recursively enumerable language.

       T   F

Let G be a context-free grammar. Then there exists a regular grammar G' such that L(G) = L(G').

       T   F

2) (20 points) Give a DFA that accepts the set of all strings of 0's and 1's that contain either the substring 10 or the substring 01, or both. For example, accepted strings should include 01, 10, 010, 101, 0110, 0000101011 and 1110101. Rejected strings include 1, 00, 111 and ε.

3) (20 points) State the pumping lemma for context-free languages.

4) Give a context-free grammar for each of the following.

(a) (10 points) The set of all strings of 0's and 1's that contain an even number of 0's.

(b) (10 points) The set of all strings of 0's and 1's that contain both an even number of 0's and an even number of 1's. For example, strings in the language include ε, 00, 11, 010100, 1010110011 and 011011. Strings not in the language include 01, 10, 001, 010, 10111, 10101 and 1110110.

5) (20 points) Prove that the recursively enumerable languages are closed with respect to set difference. In other words, if L1 and L2 are recursively enumerable languages, then so is L1 - L2. For this question be sure to explain any diagram you provide as part of your proof. In other words, simply giving a diagram with no explanation is not sufficient.