

**Formal Languages  
Comprehensive Exam  
Spring 2003**

1) (25 points)

(a) Explain one technique for showing that a language is regular. In other words, suppose you are given a language  $L$ . How do you show that  $L$  is regular?

(b) Explain one technique for showing that a language is context-free. In other words, suppose you are given a language  $L$ . How do you show that  $L$  is context-free?

(c) Explain one technique for showing that a language is recursive. In other words, suppose you are given a language  $L$ . How do you show that  $L$  is recursive?

(d) Explain one technique for showing that a language is recursively enumerable. In other words, suppose you are given a language  $L$ . How do you show that  $L$  is recursively enumerable?

(e) Suppose you are given a language  $L$  that you are told is not context-free. What proof technique(s) is/are used to prove that  $L$  is not context-free?

2) (25 points)

(a) State the pumping lemma for regular languages.

(b) What is the purpose of the pumping lemma? What is it typically used for?

3) (25 points)

(a) Prove that the regular languages are closed with respect to Kleene Star (\*). In other words, prove that if  $L$  is a regular language, then so is  $L^*$ .

(b) Prove that the context-free languages are closed with respect to Kleene Star (\*). In other words, prove that if  $L$  is a context-free language, then so is  $L^*$ .

4) (25 points) For this question you should choose either (a) or (b). Note that you may choose to answer both, in which case the best answer will be accepted.

(a) Convert the following context-free grammar  $G$  to a push-down automata  $M$  that accepts  $L(G)$ .

$S \rightarrow aABC$   
 $A \rightarrow a$   
 $B \rightarrow b$   
 $C \rightarrow cAB$   
 $C \rightarrow cC$

(b) Convert the following NFA with epsilon transitions to an equivalent NFA without epsilon transitions. Note that you are only required to show the resulting NFA.

