

Algorithms

1. Given a sequence x_1, x_2, \dots, x_n of real numbers (not necessarily positive) find a subsequence x_i, x_{i+1}, \dots, x_j (of consecutive elements) such that the sum of the numbers in it is maximum over all subsequences of consecutive elements. Your algorithm should run in linear time.
Assumption: The empty subsequence has value 0.
2. Given a list of n songs with variable integer lengths and a two-sided cassette tape with length m on each side, find a placement of the songs (or a subset of them) in the two sides of the tapes such that the empty space is minimized. Songs should be placed only once on the tape. What is the time and space complexity of your algorithm (should be polynomial)?
Hint: Use dynamic programming.
3. Describe an algorithm the minimum and maximum elements in a given sequence of n numbers, using less than $2n-3$ comparisons.
4. The input is a connected undirected graph $G = (V, E)$, a spanning tree T of G , and a vertex v . Design an algorithm to determine whether T is a valid DFS tree of G rooted at v . In other words, determine whether T can be the output of DFS under some order of the edges starting with v . The running time of the algorithm should be $O(|E| + |V|)$.
5. Let S be a set with n elements, let S_1, S_2, \dots, S_k be a collection of distinct subsets of S , each containing exactly r elements, such that $k \leq 2^{r-2}$. We want to color each element of S with one of two colors, red or blue, such that each subset S_i contains at least one red and at least one blue element. We use the following algorithm: "Take every element of S and color it either red or blue at random (with probability $1/2$), independently of the coloring of the other elements". What is the probability of failure? Based on that, how can you prove that a valid coloring always exists?

Software Engineering

1. Describe the waterfall model of the software development process, its advantages and disadvantages. What alternatives exist?
2. Using UML class diagrams, describe the objects and their relationships necessary to implement a checkout procedure in a library.
3. Create in pseudocode a procedure to merge two sorted sequences of integers with no duplicate elements into a single sorted sequence with no duplicate elements. Create the flow graph for your procedure, then evaluate McCabe's cyclomatic number of your merge procedure, which is defined as $C = e - n + 2p$, where e is the number of edges, n is the number of nodes and p is the number of connected components of the flow graph (p usually is 1).

Digital logic design

1. Prove that NAND (and NOR) gates can be used to design any circuit made of a combination of AND, OR and NOT gates.
2. Implement the following function using NAND gates: $f = (CD + D')(AB)$, where D' is the complement of D .