

**Comprehensive Exam**  
**Formal Languages**  
**Fall 2000**

This test consists of 5 questions. Read each question carefully and make sure that your answer is clear and easily identifiable.

1) (20 points) Give a DFA or an NFA for the set  $0^*1^*01$

2) (20 points) Give a context-free grammar for the set  $0^*1^*01$

3) (20 points) Prove that the regular languages are closed with respect to concatenation.

4) Consider the following context-free grammar  $G$  with starting non-terminal  $A$ .

$$A \rightarrow AB \quad (1)$$

$$A \rightarrow AA \quad (2)$$

$$B \rightarrow BB \quad (3)$$

$$B \rightarrow C \quad (4)$$

$$A \rightarrow a \quad (5)$$

$$B \rightarrow b \quad (6)$$

$$C \rightarrow c \quad (7)$$

Note that the CFG given above has 3 nonterminals. In addition, note that the string  $x = aaaabbbc$ , which can be generated by the grammar, has length  $2^3 = 8$ .

(a) (10 points) Using the above grammar, give a parse tree for the string  $x = aaaabbbc$ .

The pumping lemma tells us that this string can be broken into five parts  $u$ ,  $v$ ,  $w$ ,  $x$ , and  $y$ , such that  $x = uvwxy$ ,  $1 \leq |vwx| \leq n$ ,  $1 \leq |vx|$ , and  $uv^iwx^iy \in L(M)$ , for all  $i \geq 0$ .

(b) (10 points) Based on your answer to part (a), show how to break the string  $x = aaaabbbc$  into strings  $u$ ,  $v$ ,  $w$ ,  $x$ , and  $y$ .

$u =$

$v =$

$w =$

$x =$

$y =$

5) (20 points) For each of the following, specify whether the given language is recursive, recursively enumerable but not recursive, or not recursively enumerable.

(a)  $L = \{x \mid x \in \{0, 1\}^* \text{ and } x \text{ encodes a DFA}\}$

(b)  $L = \{x \mid x \in \{0, 1\}^* \text{ and } x \text{ encodes a Turing machine}\}$

(c)  $L = \{x \mid x \in \{0, 1\}^*, x = \langle M, w \rangle, M \text{ is a DFA and } w \in L(M)\}$

(d)  $L = \{x \mid x \in \{0, 1\}^*, x = \langle M, w \rangle, M \text{ is a DFA and } w \notin L(M)\}$

(e)  $L = \{x \mid x \in \{0, 1\}^*, x = \langle M, w \rangle, \text{ where } M \text{ is a TM and } w \in L(M)\}$