Computer Science
Comprehensive Examination
Formal Languages
Fall 2004

1) (20 points) Let $T=(V,E)$ be a tree with $|V|>=1$. Then T is a *strict binary tree* if and only if each vertex in T is either a leaf or has exactly two children. In addition, let $L(T)$ denote the number of leaves in T.

Suppose you want to prove that $L(T) <= 2^h$, where h is the height of T (note: the height of a tree with one node is 0, the height of a tree with two levels is 1, etc).

(a) What proof technique would you use to prove this theorem?

(b) Outline the proof technique, in general terms, given in part (a).

(c) Prove the theorem (i.e., $L(T) <= 2^h$) using the proof technique specified in part (a) following the outline specified in part (b).

2) (15 points) Give a formal definition of a turing machine.

3) (20 points) If possible, give a context-free grammar for each of the following languages. If you cannot give a context-free grammar for one of the languages then explain why.

(a) {x | x is a string of 0's and 1's that begins with 101}

(b) {x | x is a string of 0's, 1's and 2's that begins with 111, ends with 222, and contains at least one occurrence of 000}

(c) {x | x is of the form $0^i1^i2^i3^i$, where i >= 0}

4) (20 points)

(a) State the pumping lemma for regular languages.

(b) Prove that the language 0^i1^i, where $i>=0$, is not regular.