

**Formal Languages
Comprehensive Exam
Fall 2003**

1) (20 points)

(a) Give a DFA for the set of all strings of 0's and 1's that contain an even number of 0's.

(b) Give a context-free grammar for the set of all strings of 0's and 1's of the form $0^i 1^{2i}$ where $i \geq 0$.

(c) Give a regular expression for the set of all strings of 0's and 1's that contain at least three 1's.

(d) Is the language described in part (a) regular (yes or no)? Is the language described in part (b) regular (yes or no)? Is the language described in part (c) regular (yes or no)? Note that for this question, no proof or explanation is required, although you can provide one if you like.

2) (25 points)

(a) State the pumping lemma for context-free languages.

(b) The pumping lemma for context-free languages dictates that any string of sufficient length from a context-free language can be broken up into several parts. Explain where those parts come from. In other words, outline the part of the proof of the pumping lemma that shows how to break the very long string into substrings.

3) (20 points)

(a) Give a formal definition of a regular expression.

(b) Give a formal definition of a deterministic turing machine.

4) (15 points) Which of the following turing machine models is considered to be the most powerful? Be sure to explain your answer. In particular, be sure to explain what you mean by “powerful.”

Deterministic one-tape turing machine
Non-deterministic, multi-dimensional, multi-tape turing machine
Non-deterministic one-tape turing machine
Deterministic, multi-dimensional, multi-tape turing machine
Deterministic multi-tape turing machine
Non-deterministic multi-tape turing machine

5) (20 points) Let L_1 be a recursively enumerable language, and let L_2 be a context-free language. Prove that the intersection of L_1 and L_2 is a recursively enumerable language.