Discrete Mathematics Comprehensive Examination

Department of Computer Sciences, College of Engineering,
Florida Institute of Technology

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Sign the exam with your student number — not your name. Each of the five sections are equally weighted.

1. Induction

Use mathematical induction to prove that, for all $n \in \{1, 2, 3, \ldots\}$,

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$$

2. Combinatorics

An ice cream parlor has 28 diffrent flavors of ice cream, 8 diffrent sauces and 12 different toppings. How many diffrent sundaes can be made if a sundae contains 3 scoops of ice cream, where each flavor can be used more than once and the order of the scoops does not matter; 2 kinds of sauce where each sauce can be used only once and the order of sauces does not matter; and 3 toppings, where each topping can be used only once and the order of the toppings does not matter?

3. Logarithms

1. How many bits does it take to encode 1,000,000 different values?

2. If we have an alphabetically sorted list of 128 names, how many records do we need to look at to find a given individual?

3. Is it `true` that $10^{100} < 2^{256}$? Use the approximation that $\log_2 10 \approx 3.322$.

4. Summations

1. What is the formula for the sum of the first n natural numbers?

$$\sum_{k=0}^{n-1} k = 0 + 1 + 2 + \cdots + (n-1)$$

2. What is the formula for the sum of the first n powers of 2?

$$\sum_{k=0}^{n-1} 2^k = 2^0 + 2^1 + 2^2 + \cdots + 2^{n-1}$$

3. What is the formula for the sum of the binomial coefficients?

$$\sum_{k=0}^{n-1} \binom{n-1}{k} = \binom{n-1}{0} + \binom{n-1}{1} + \cdots + \binom{n-1}{n-1}$$

4. What is the formula for the sum of the first n Fibonacci numbers?

$$\sum_{k=0}^{n-1} F_k = 0 + 1 + 1 + \cdots + F_{n-1}$$

5. Relations

Consider the relation $(a, b) \sim (c, d)$ if and only if $ad = bc$ on ordered pairs of natural numbers.

1. Give an example of values for a, b, c, and d such that $(a, b) \sim (c, d)$.

2. Is the relation reflexive? Explain your answer.

3. Is the relation symmetric? Explain your answer.

4. Is the relation antisymmetric? Explain your answer.

5. Is the relation transitive? Explain your answer.