

**Computer Science
Comprehensive Examination
Formal Languages
Spring 2005**

This test consists of five questions. You may choose any four. Note that if you answer all five questions, then your four best answers will be used to compute your grade.

1) (25 points) Draw a diagram showing the hierarchy of languages (commonly referred to as the *Chomsky Hierarchy*).

2) (25 points) Give the formal definition for a regular expression.

3) (25 points) Give a regular expression for each of the following languages.

(a) $\{x \mid x \in \{0, 1\}^* \text{ and } x \text{ begins with } 101\}$

(b) $\{x \mid x \in \{0, 1, 2\}^* \text{ and } x \text{ begins with } 111, \text{ ends with } 222, \text{ and contains at least one occurrence of } 000\}$

(c) $\{x \mid x \in \{0, 1\}^* \text{ and every occurrence of } 0 \text{ is followed by a } 1\}$

4) (25 points) Prove that the regular languages are closed with respect to union. In other words, if L_1 and L_2 are regular languages, then so is $L_1 \cup L_2$.

5) (25 points) Define each of the following terms or phrases.

(a) bijection

(b) cardinality (hint: two sets A and B have the same cardinality if...)

(c) countably infinite (hint: a set S is said to be countably infinite if...)

(d) countable (hint: a set S is said to be countable if...)