Formal Languages
Comprehensive Exam
Spring 2007

1) (20 points)

(a) Give a DFA for the set of all strings of 0's and 1's that start with a 1 and end with a 1. Strings in the language include 1, 101, 111, 1001101, etc.

(b) Give a context-free grammar for the set of all strings of 0's and 1's of the form 0^i1^j where j>i (Hint: try 0^n1^n first).

(c) Give a regular expression for the set of all strings of 0's and 1's that contain at least one occurrence of the substring 00, and at least one occurrence of the substring 11. For example, strings in the language include 101001011001, 1100, 00111, etc.

(d) Is the language described in part (a) regular (yes or no)? Is the language described in part (b) regular (yes or no)? Is the language described in part (c) regular (yes or no)? Note that for this question, no proof or explanation is required, although you can provide one if you like.

2) (25 points)

(a) State the pumping lemma for context-free languages.

(b) The pumping lemma for context-free languages dictates that any string of sufficient length from a context-free language can be broken up into several parts. Explain where those parts come from. In other words, outline the part of the proof of the pumping lemma that shows how to break the very long string into substrings.

3) (20 points)

(a) Give a formal definition of a PDA.

(b) Suppose we wanted to define a new type of PDA that had two stacks instead of just one. For this question you are required to give a formal definition for such a machine.

4) (15 points) Which of the following turing machine models is considered to be the most powerful? Be sure to explain your answer. In particular, be sure to explain what you mean by "powerful."

Deterministic one-tape turing machine
Non-deterministic, multi-dimensional, multi-tape turing machine
Non-deterministic one-tape turing machine
Deterministic, multi-dimensional, multi-tape turing machine
Deterministic multi-tape turing machine
Non-deterministic multi-tape turing machine

5) (20 points) Let L1 be a context-free language, let L2 be a regular language, and let L3 be a recursive language. Prove that the intersection of L1, L2 and L3 is a recursively enumerable language. Note that for this question you may make use of known closure properties.