

**Computer Science
Comprehensive Examination
Formal Languages
Fall 2005**

1) (20 points) Give the formal definition of a PDA.

2) (20 points) Give a DFA for each of the following languages.

(a) $\{x \mid x \in \{0, 1\}^* \text{ and } x \text{ begins with } 10\}$

(b) $\{x \mid x \in \{0, 1\}^* \text{ and every consecutive sequence of two symbols contains at least one } 0\}$. For example, strings in the language include ϵ , 0, 1, 01, 10, 00, 010, 100, 101, 0101010, and strings not in the language include 11, 110, 011, 0011, 0110, and 11000110.

3) (20 points) Prove the recursively enumerable languages are closed with respect to intersection. In other words, if L_1 and L_2 are recursively enumerable languages, then so is $L_1 \cap L_2$.

4) (20 points) Give a formal definition for each of the following terms or phrases.

(a) language

(b) regular language

(c) context-free language

(d) recursive language

(e) recursively enumerable language

5) (20 points) Recall that a context-free grammar is said to be a regular grammar if each production has one of the following three forms:

$A \rightarrow aB$

$A \rightarrow a$

$A \rightarrow \epsilon$

Where A and B are any non-terminals, and a is any terminal. Prove that a language L is a regular language if and only if there exists a regular grammar G such that $L = L(G)$ (Hint: use DFAs).