

**Computer Science
Comprehensive Examination
Formal Languages
Spring 2001**

1) Give a regular expression for the following language:

$\{x \mid x \text{ is a string of zeros and ones that begins with } 01 \text{ and ends with } 10\}$

2) Let $L = \{x \mid x \text{ is a string of the form } 0^n 1^{3n} \text{ where } n > 0\}$

(a) Give a context-free grammar for L .

(b) Is your answer in part (a) a regular grammar (yes or no)? If not, then explain why not.

(c) Is the language L regular (yes or no)? For this question, you are *not* required to give a formal proof. However, a brief explanation of your answer is required.

3) For each of the following, let L_1 be a regular language, and let L_2 be a context-free language.

(a) Is $L_1 \cup L_2$ guaranteed to be a *context-free* language (yes or no)? If yes, then explain why, and if not then provide a counter-example, i.e, give a regular language L_1 and a context-free language L_2 such that $L_1 \cup L_2$ is not context-free.

(b) Is $L_1 \cup L_2$ guaranteed to be a *regular* language (yes or no)? If yes, then explain why, and if not then provide a counter-example, i.e, give a regular language L_1 and a context-free language L_2 such that $L_1 \cup L_2$ is not regular.

4) For each of the following let M be a Turing machine. In each case be sure to explain your answer.

(a) Is $L(M)$ recursively enumerable (yes or no)?

(b) Is $L(M)$ guaranteed to be recursive (yes or no)?

(c) Is $\overline{L(M)}$ guaranteed to be recursively enumerable (yes or no)?

(d) Is $\overline{L(M)}$ guaranteed to be recursive (yes or no)?

(e) Is $L(M) \cup \overline{L(M)}$ guaranteed to be recursive (yes or no)?