The Four Color problem is one of the most famous problems in Mathematics. The problem consists of the question whether any map can be colored using four colors in such a way that adjacent regions (i.e. those sharing a common boundary segment, not just a point) receive different colors. This problem has been formulated first by Francis Guthrie in 1852 (published in 1878) and was unsolved for roughly a century. Wolfgang Haken und Kenneth Appel could prove the four color theorem with the help of a computer program in 1977 [Ken77].

**Exercise 4.1** [4 points]
Read chapter 4.1 – 4.2 and 6.2 of the textbook.

1. Define in your own words the terms constraint satisfaction problem, constraint, back-tracking search, back-jumping, and min-conflicts. [2.5 points]

2. How many solutions are there for the three-color map-coloring problem in Figure 1? Elaborate your answer. [1.5 points]

![States and Territories](image1.png)

![Constraint-graph](image2.png)

Figure 1: a) The principal states and territories of Australia. b) The map-coloring problem represented as a constraint graph.
Exercise 4.2 [16 points]
Consider the political map of the South-Eastern states of the US (states North Carolina, South Carolina, Virginia, Tennessee, Kentucky, West Virginia, Georgia, Alabama, Mississippi, and Florida, see Figure 2). How can we color this map with the four color theorem using a Genetic Algorithm? You can also consider all 51 states of the USA (Washington DC is counted as a state in this case).

1. How is this problem represented in general (write in your own words). Define the states, the goal-test, and the successor function of your problem. [2 points]

2. Implement your algorithm and show the results. You may want to use the framework provided by Alexander Härzl (download Java, C++, C). Fan Zhang has created a Python Framework which can be found here: Python. Neighboring states can be found in text files provided by Andreas Seekircher us_states_10_ij.txt and us_states_51_ij.txt [14 points]

References