

CSC527: Homework #1: Fundamentals

Question I (10 points)

Suppose that A and B are nonempty sets and $A \times B \subseteq B \times A$. Show that $A = B$. Suggestion: show that $A \subseteq B$ and $B \subseteq A$, using proof by contradiction in each case.

Question II (10 points each)

Each case below give a relation on the set of all nonempty subsets of $\mathcal{N}$. In each case, say whether the relation is reflexive, whether it is symmetric, and whether it is transitive.

- a. R is defined by: ARB if and only if $A \subseteq B$.
- b. R is defined by: ARB if and only if $A \cap B \neq \emptyset$.
- c. R is defined by: ARB if and only if $1 \in A \cap B$.

Question III (10 points)

Prove using mathematical induction that for every nonnegative integer n ,

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

Question IV (5 points each)

In each case below, construct a truth table for the statement and find another statement with at most one operator ($\vee$, $\wedge$, $\neg$, or $\rightarrow$) that is logically equivalent.

- a. $(p \rightarrow q) \wedge (p \rightarrow \neg q)$
- b. $p \wedge (p \rightarrow q)$
- c. $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
- d. $p \leftrightarrow (p \leftrightarrow q)$
- e. $q \wedge (p \rightarrow q)$