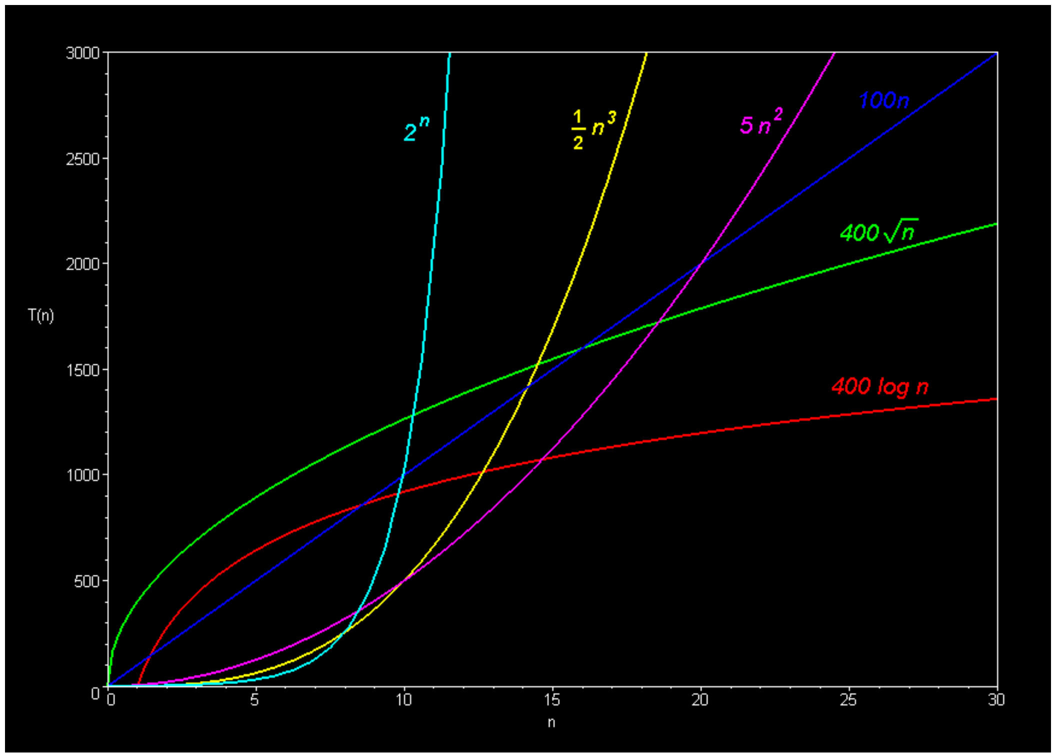
Data Structures and Algorithm Analysis (CSC317)



Week 2: Growth of Functions (cont)

Picture from
http://science.slc.edu/~jmarshall/courses/2002/spring/cs50/BigO/

Summary Oh, Omega, Theta

Oh

- $O(n)$ asymptotic upper, like $\leq$ $\quad f(n) \leq cg(n)$

Omega

- $\Omega(n)$ asymptotic lower, like $\geq$ $\quad f(n) \geq cg(n)$

Theta

- $\Theta(n)$ asymptotic tight, like $=$ $\quad c_1 g(n) \leq f(n) \leq c_2 g(n)$

More on Oh, Omega, Theta

Theorem: $f(n) = \theta(n)$

if and only if (iff)

More on Oh, Omega, Theta

Theorem: $f(n) = \theta(n)$

if and only if (iff)

$f(n) = O(n)$ and $f(n) = \Omega(n)$

More on Oh, Omega, Theta

Example: Is $f(n) = n^2 + 5 \quad \Omega(n^3)$?

More on Oh, Omega, Theta

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(Is f(n) bounded below by?)

Answer: no.

Proof by contradiction: Suppose $f(n) = \Omega(n^3)$

More on Oh, Omega, Theta

Example: Is $f(n) = n^2 + 5 \quad \Omega(n^3)$?

(Is f(n) bounded below by?)

Answer: no.

Proof by contradiction: Suppose $f(n) = \Omega(n^3)$

Then there exist $n_0; c$ such that for all : $n \geq n_0$

$$n^2 + 5 \geq cn^3;$$

$$n^2 + 5n^2 \geq cn^3;$$

$$6n^2 \geq cn^3;$$

$$6 \geq cn$$

Can't be true for all $n \geq n_0$

Some properties of Oh, Omega, Theta

Transitivity: $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$

then $f(n) = \Theta(h(n))$

(same for $\Omega; O$??)

Some properties of Oh, Omega, Theta

Reflexivity: $f(n) = \Theta(f(n))$

(same for $\Omega; O$??)

Some properties of Oh, Omega, Theta

Symmetry: $f(n) = \Theta(g(n))$

iff $g(n) = \Theta(f(n))$

(same for $\Omega; O$??)

Some properties of Oh, Omega, Theta

Symmetry: $f(n) = \Theta(g(n))$

iff $g(n) = \Theta(f(n))$

(same for $\Omega; O$??) No. Only for Θ (why?)

Summary: properties of Oh, Omega, Theta

Transitivity: $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n))$

then $f(n) = \Theta(h(n))$

(same for $\Omega; O$)

Reflexivity: $f(n) = \Theta(f(n))$

(same for $\Omega; O$)

Symmetry: $f(n) = \Theta(g(n))$

iff $g(n) = \Theta(f(n))$

(only for Θ)

Final thoughts: Oh, Omega, Theta

- Useful comparison formula of functions in book (section 3.2)