Practical Proof Checking for Program Certification

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Abstract

Program certification aims to provide explicit evidence that a program meets a specified level of safety. This evidence must be independently reproducible and verifiable. We have developed a system, based on theorem proving, that generates proofs that auto-generated aerospace code adheres to a number of safety policies. For certification purposes, these proofs need to be verified by a proof checker. Here, we describe and evaluate a semantic derivation verification approach to proof checking. The evaluation is based on 109 safety obligations that are attempted by EP and SPASS. Our system is able to verify 129 out of the 131 proofs found by the two provers. Most proofs are checked completely in less than 30 seconds wall clock time, This shows that the proof checking task arising from a substantial prover application is practically tractable.

1 Introduction

Program certification tries to show that a given program achieves a certain level of quality, safety, or security. Its result is a *certificate*, i.e., independently checkable evidence of the properties claimed. Certification approaches vary widely, ranging from code reviews to full formal verification. The highest degree of confidence is achieved with approaches that are based on formal methods, and use logic and theorem proving to construct the certificates.

Over the last few years we have developed, implemented, and evaluated a certification approach that uses Hoare-style techniques to formally demonstrate the safety of aerospace programs that are automatically generated from high-level specifications [WSF02a, WSF02b, DFS04c, DFS04b, DFS05]. In that work, we have extended a code generator so that it simultaneously generates code *and* the detailed annotations, e.g., loop invariants, that enable fully automated safety proofs. A verification condition generator (VCG) processes the annotated code and produces a set of *safety obligations* that are provable if and only if the code is safe. An automated theorem prover (ATP) discharges these obligations and its proofs serve as certificates; we focus on *automated*—as opposed to interactive or (the auto-modes of) tactic-based—provers, since we are aiming at a fully automated "push-button" tool.

For certification purposes, users and certification authorities like the FAA must be assured—or better yet, given explicit evidence—that none of the individual tool components yield incorrect results and, hence, that the certificates are valid. The assurance can take a variety of different forms, e.g., tool pedigree, code inspections, paper-and-pencil proofs, or result checking. In this paper, we focus on automatically checking the correctness of the proofs generated by the ATP, which are crucial elements in our certification chain.

Proof checking is of course not necessary if the applied ATP is known to be correct. However, program certification is a difficult task that requires substantial "deductive power": the longest proof found during experiments involved more than 8000 inference steps. Consequently, simple "correct-by-inspection" theorem provers like leanTAP [BP95], or tactic-based provers built on top of a trusted kernel like Isabelle [Pau89], are not powerful enough.[1] Instead, we need to employ high-performance ATPs, which use complicated calculi, elaborate data structures, and optimized implementations. This makes formal verification of their correctness infeasible [MSM00]. One could argue that these provers have been extensively validated by the theorem proving community (e.g., the soundness checks required for participation in the CASC), so that a formal verification is not necessary. However, this tool pedigree argument is weak. Most ATPs are under continuous development and single versions are never subjected to enough validation to achieve sufficient "social validation."[2] Moreover, the validation is necessarily incomplete. There have been several published instances of (unintentional) unsoundness in ATPs participating in the CASC, which have been detected only afterwards [SS99, Sut00b, Sut05].

As an alternative to formally verifying or extensively validating the ATPs, they can be extended to generate sufficiently detailed proofs that can be independently verified by a *proof checker*. The checker's function is to verify that the ATP's output is really a proof in the logical system in use. There are several approaches to proof checking, including syntactic validation of proof steps (as in [MSM00]), higher-order proof term reconstruction [BN00], higher-order proof step checking [Won99], reducing proof checking to type checking (as in Coq [BC04]), and semantic derivation verification [SB05]. Semantic derivation verification has been used in this work. In semantic derivation verification, the required semantic properties of each proof step are encoded in a proof check obligation (typically an implication from the premises of the applied inference rule to its conclusion), which is then discharged by a trusted ATP. This way, the trusted ATP verifies the proof output of the original ATP. This approach is tractable because the correctness proof for each individual step in the original proof is substantially easier than the original proof itself, and thus within reach of the trusted ATP. For certification purposes, all proofs found by the trusted ATP become part of the certificate that is delivered by the overall certification system.

This paper describes how a semantic derivation verifier has been used to check the proofs that are found by ATPs for the safety obligations generated in the program cer-

[1] See `http://www.cl.cam.ac.uk/users/jeh1004/software/metis/performance.html` for benchmark data.

[2] The notable exception is Otter [McC03b], which has been essentially unchanged since 1996. However, previous experiments have shown that its performance is not sufficient for discharging the safety obligations we generate [DFS05].

tification process. The success of ATPs in discharging the safety obligations has been described in [DFS04a]. The success of (trusted) ATPs in verifying the resultant proofs is demonstrated here. Section 2 provides the necessary background on the program certification process, and Section 3 describes the semantic verification technique. Sections 4 and 5 provide empirical data that illustrate the success of the approach. Section 6 concludes, and discusses directions for future work.

2 Formal Program Certification

Formal program certification is based on the idea that the mathematical proof of some program property can be regarded as an externally verifiable certificate of this property. It is a limited variant of full program verification because it proves only individual properties and not the complete behavior, but it uses the same underlying technology.

2.1 Safety Policies

Formal program certification ensures that a program complies with a given *safety policy*. This is a formal characterization that the program does not "go wrong", i.e., does not violate certain conditions. A safety policy is defined by a set of Hoare-style inference rules and auxiliary definitions. The formal basis of this approach is explored in [DF03].

Safety policies exist at two levels of granularity. *Language-specific* policies can be expressed in terms of the constructs of the underlying programming language itself. They are sensible for any given program written in the language, regardless of the application domain. Typical examples of language-specific policies are array-bounds safety (i.e., each access to an array element to be within the specified upper and lower bounds of the array) and variable initialization-before-use (i.e., each variable or individual array element has been assigned a defined value before it is used). Various coding standards (e.g., restrictions on the use of loop indices) also fall into this category. *Domain-specific* properties are, in contrast, specific to a particular application domain and not applicable to all programs. These typically relate to high-level concepts outside the language. In principle, they are independent of the target programming language although, in practice, they tend to be be expressed in terms of program fragments. A typical example is matrix symmetry which requires certain two-dimensional arrays to be symmetric.

2.2 Generating Safety Obligations

For certification purposes, code must be *annotated* with information relevant to the selected safety policy. The annotations contain local information in the form of logical pre- and post-conditions and loop invariants, which is then propagated through the code. The fully annotated code is then processed by a verification condition generator (VCG), which applies the rules of the safety policy to the annotated code in order to generate the safety conditions. As usual, the VCG works backwards through the code, and safety conditions are generated at each line. Our VCG has been designed to be "correct-by-inspection", i.e., to be sufficiently simple that it is straightforward to see that it correctly implements the rules of the logic. Hence, the VCG does not implement any optimizations, such as structure sharing on verification conditions or even apply any simplifications. Consequently, the generated verification conditions tend to be large and must be simplified. The more manageable simplified verification conditions can then processed by an ATP.

2.3 Certifiable Program Synthesis

As usual in Hoare-based approaches, the annotation effort can quickly become overwhelming and constitute a barrier for the adoption of the technique. This can be overcome by a *certifiable program synthesis system* that automatically generates the code *and* the detailed annotations from a high-level specification of the problem. The basic idea is to make the annotations part of the code templates so that they can be instantiated and refined in parallel with the code fragments. We have implemented this approach in two synthesis systems, AutoFilter [WS04], which generates state estimation code based on the Kalman filter algorithm, and AutoBayes [FS03], which generates statistical data analysis code.

Figure 1 shows the overall architecture of a certifiable program synthesis system. At its core is the original synthesis system that generates code for a given specification. The core system is extended for certification purposes (i.e., by the annotation templates), and augmented with a VCG, a simplifier, an ATP, and a proof checker. These components are described in more detail in [DF03, DFS04b, DFS04c].

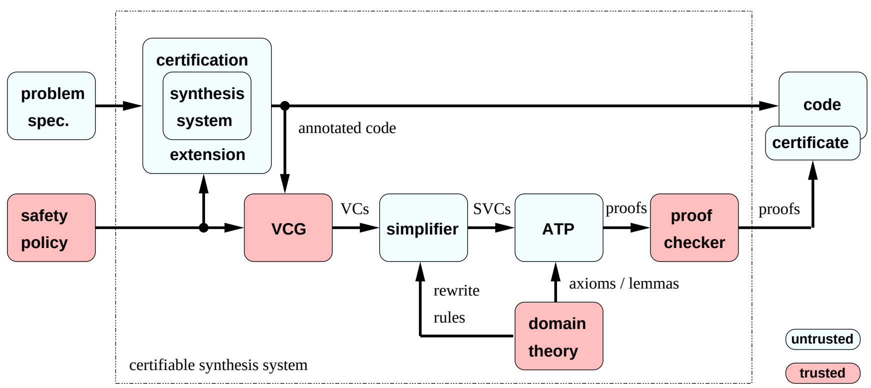

Figure 1: Certifiable program synthesis: System architecture

Similar to proof carrying code [NL98], the architecture distinguishes between trusted and untrusted components, shown in Figure 1 in red (dark grey) and blue (light grey), respectively. Components are called *trusted*—and must thus be correct—if any errors in them can compromise the assurance provided by the overall system. *Untrusted* components, on the other hand, are not crucial to the assurance because their results are double-checked by at least one trusted component. In particular, the correctness of the certifiable program synthesis system does not depend on the correctness of its two largest components: the original synthesis system (including the certification extensions), and the ATP; instead, we need only trust the safety policy, the VCG, and the proof checker.

3 Semantic Derivation Verification

The proofs produced by ATP systems can be considered more abstractly as derivations. For our purposes, a *derivation* is a directed acyclic graph (DAG), whose leaf nodes are formulae (possibly derived) from the input problem, whose interior nodes are formulae inferred from parent formulae, and whose unique root node is the final derived formula. In semantic derivation verification, the required semantic properties of each formula in a derivation are encoded in a separate *obligation*. This is then *discharged* by a trusted ATP.

Derivation verification involves three notionally distinct phases. First, it is necessary to check the overall structure of the derivation. This ensures that the ATP output actually is a well-formed derivation DAG. Second, it is necessary to check that each leaf node is a formula that occurs in, or is derived from, the input problem. This ensures that the ATP actually solves the original problem. Third, it is necessary to check that each inferred formula has the required semantic relationship to its parents. This finally ensures that the proof is correct. The required semantic relationship of an inferred formula to its parents depends on the intent of the inference rule used. Most commonly an inferred formula is intended to be a logical consequence of its parents, but in other cases, e.g., Skolemization and splitting, the inferred formula has a weaker link to its parents. A comprehensive list of inferred formula statuses is given in [SZS04].

The main advantage of semantic derivation verification over other approaches is that it decouples proof checking from the production of the proof—any ATP can serve as the trusted system that checks the output from the untrusted production system. Moreover, the approach is independent of the particular inference rules used in the production ATP, and is also robust with respect to any preprocessing of the input formulae that the production ATP might perform (since it checks the relationship between the original goal and the leaves irrespective of the intermediate steps).

3.1 Logical Consequences and Relevance

The basic technique for verifying logical consequences is well known and quite simple. The earliest use appears to have been in the in-house verifier for SPASS [WBH$^+$02]. For each inference of a logical consequence in a derivation, a *theorem obligation* is formed; this formalizes that the inferred formula follows from the parent formulae. If the inference rule implements any theory (e.g., paramodulation implements most of equality theory), then the corresponding axioms of the theory are added as axioms of the obligation. The obligation is then handed to the trusted ATP system. If the trusted system solves the problem (i.e., finds a proof), the obligation has been discharged.

This verification of logical consequences ensures the soundness of the inference steps, but does not check for *relevance* [AB75]. As a contradiction in first order logic entails everything, an inference step with contradictory parents can soundly infer anything. If such inferences should be rejected[3], a *satisfiability obligation*, consisting of only axioms, is formed from the parents of the inference. This must be discharged by being shown to be satisfiable. Due to the semi-decidability of first order logic, such satisfiability obli-

[3]This not not always the case, e.g., parents that include a clause that is derived from the negation of the conjecture may correctly be unsatisfiable.

gations cannot be guaranteed to be discharged. Three alternative techniques, described here in order of preference, may be used to show satisfiability. First, a finite model of the axioms may be found using a model generation system such as MACE [McC03a] or Paradox [CS03]. Second, a saturation of the axioms may be found using a saturating system such as SPASS or E [Sch02b]. Third, an attempt to show the axioms to be contradictory can be made using a refutation system. If this attempt succeeds then the obligation cannot be discharged, but if it fails it provides an incomplete assurance that the formulae are satisfiable.

3.2 Splitting

Many contemporary ATPs that build refutations for CNF problems use *splitting*. Splitting reduces a CNF problem to one or more potentially easier problems by dividing a clause into two subclauses. There are several variants of splitting that have been implemented in specific ATPs, including *explicit splitting* as implemented in SPASS, and forms of *pseudo-splitting* as implemented in Vampire [RV01] and E. Verification of splitting inferences requires several theorem obligations to be discharged.

Explicit splitting takes a CNF problem $S \cup \{L \vee R\}$, in which L and R do not share any variables, and replaces it by two subproblems $S \cup \{L\}$ and $S \cup \{R\}$. If both the subproblems have refutations (i.e., are unsatisfiable), then it is assured that the original problem is unsatisfiable. To verify a explicit splitting step's role in establishing the overall unsatisfiability of the original problem clauses, a theorem obligation to prove $\neg(L \vee R)$ from $\{\neg L, \neg R\}$ must be discharged.

Pseudo-splitting takes a CNF problem $S \cup \{L \vee R\}$, in which L and R do not share any variables, and replaces $\{L \vee R\}$ by either (i) $\{L \vee t, \neg t \vee R\}$, or (ii) $\{L \vee t_1, R \vee t_2, \neg t_1 \vee \neg t_2\}$, where t and t_i are new propositional symbols. Vampire implements pseudo-splitting by (i) and E implements it by (ii). The replacement does not change the satisfiability of the clause set—any model of the original clause set can be extended to a model of the modified clause set, and any model of the modified clause set satisfies the original one [RV01, Sch02a]. The underlying effect of (i) is to introduce a new definitional axiom, $t \Leftrightarrow \neg \forall L$, and of (ii) is to introduce two new definitional axioms, $t_1 \Leftrightarrow \neg \forall L$ and $t_2 \Leftrightarrow \neg \forall R$. Pseudo-splitting steps are verified by discharging theorem obligations that prove (i) the split clause from the replacement clauses, and (ii) each of the replacement clauses from the split clause and the new definitional axiom(s).

3.3 Leaf Formulae

The leaves of a derivation must occur in or be derived from the original problem—otherwise, the ATP solves a different problem. To verify this, theorem obligations to prove each leaf formula from the input formulae must be discharged. An advantage of the semantic technique for verifying leaf formulae is that it is somewhat robust to preprocessing inferences that are performed by some ATP systems. For example, Gandalf [Tam98] may factor and simplify input clauses before storing them in its clause data structure. The leaves of refutations output by Gandalf may thus be derived from input clauses, rather than directly being input clauses. These leaves are logical consequences of the original input clauses, and can be verified using this technique.

If the input problem is in FOF, and the derivation is a CNF refutation, the leaf clauses may have been formed with the use of Skolemization. Such leaf clauses are not logical consequences of the FOF input formulae. Skolemization steps can be incompletely verified by discharging a theorem obligation to prove the parent formula from the Skolemized formula.

3.4 Structural Verification

All forms of proof checking also include, at least implicitly, some *structural verification*. Structural verification checks that inferences have been used correctly in the context of the overall derivation.

For all derivations, two structural checks are necessary: First, the specified parents of each inference step must exist in the derivation. When semantic verification is used to verify each inference step then the formation of the obligation problems relies on the existence of the parents, and thus performs this check. The check can also be done explicitly. Second, there must not be any loops in the derivation. For derivations that claim to be CNF refutations, it is necessary to also check that the empty clause has been derived.

For refutations that use explicit splitting, two further structural checks are necessary. First, it is necessary to check that both subproblems have been refuted. Second, it is necessary to check that L (R) and its descendants are not used in the refutation of the R (L) subproblem. For refutations that use pseudo-splitting, a structural check is required to ensure that the "new propositional symbols" really are new, and not used elsewhere in the refutation.

3.5 Implementation

The semantic verification techniques described here have been implemented in the GDV system. GDV is implemented in C, using the JJParser library for input, output, and data structure support. The inputs to GDV are a derivation in TPTP format [SZS04], the original problem in TPTP format, a set of trusted ATPs to discharge the theorem obligations, and a CPU time limit for the trusted ATPs for each obligation. SystemOnTPTP [Sut00a] is used to run the trusted ATPs. Obligations that are successfully discharged are reported, and the output from the discharging is optionally retained for later inspection. If an obligation cannot be discharged, or a structural check fails, GDV reports the failure.

4 Experimental Setup

In [DFS05], we evaluate multiple ATPs on 366 safety obligations generated from the certification of programs generated by the AutoBayes and AutoFilter program synthesis systems. Of those 366 problems, 109 were selected for inclusion in the TPTP problem library [SSRL], the standard library of test problem for testing and evaluating ATPs. The 109 problems were selected based on the results of evaluating several state-of-the-art ATPs against the problems, and were selected so as to be "difficult", i.e., with TPTP difficulty ratings strictly between 0.0 and 1.0 [SS01].

As a practical test and evaluation of the proof checking approach described in this paper, we scrutinized the proofs generated for these 109 problems by the ATPs EP [Sch02b] (Version 0.82) and SPASS [WBH+02] (Version 2.1). Both EP and SPASS work by converting the axioms and the negated conjecture to CNF, and then using clausal reasoning to find a refutation. Both systems are based on the superposition calculus, but differ in the specific inference rules used. A notable difference is EP's use of pseudo-splitting and SPASS's use of explicit splitting. Additionally, the systems have quite different control heuristics. As a result the proofs produced by the two systems have quite different characteristics. The proofs output by EP include details of the FOF to CNF conversion, and the subsequent CNF refutation. The proofs are natively output in TPTP format. The proofs output by SPASS document the CNF refutation, but not the FOF to CNF conversion. The SPASS proofs are natively in DFG format, which is translated to TPTP format prior to verification.

The verification was done using the GDV system. For the verification of the EP proofs, GDV was configured to verify all aspects of each proof: leaves were verified as being (derived) from the input problem, all inferred formulae were semantically verified with relevance checking, all splitting steps were verified, and the derivation was structurally verified. For the verification of the SPASS proofs, GDV was configured to verify selected aspects of the proof. Leaves could not be verified because SPASS does not document the FOF to CNF conversion, all inferred formulae are semantically verified but without relevance checking, all splitting steps were verified but the independence of the subproblems was not verified in the larger proofs because of the computation complexity, and the derivation was structurally verified (with the exception of the splitting aspect just mentioned). The trusted ATPs used were Otter 3.3 [McC03b] for discharging logical consequence obligations, Paradox 1.1 [CS03] for finding finite models, and SPASS 2.1 for finding saturations.[4] The outputs from Otter, Paradox, and SPASS were retained, and are available as part of any certificate. The verifications were done on Intel P4 2.8GHz computers with 1GB RAM, and running the Linux 2.4 operating system. The CPU time limit for each discharge was 10s.

5 Experimental Results

Out of the 109 problems, EP can solve 48 and SPASS can solve 83, thus giving a total of 131 proofs to check. The 48 problems solved by EP are a subset of those solved by SPASS, but the proofs are obviously different. Table 1 summarizes the results. The first row shows the number of problems solved out of the 109, and the second row shows how many of those were verified by GDV with the checks described above. The next two rows give the numbers of theorem obligations that were generated for the verifications and discharged by Otter, respectively. The next five rows give the average number of theorem obligations per proof and their distribution, giving an indication of the distribution of the proof sizes. The next block of four rows gives the distribution of the CPU times taken by Otter to discharge the theorem obligations. The final two rows

[4]Satisfiability tests, which employ saturation finding, are used only in the verification of leaves and relevance checking. As these checks were not done for the SPASS proofs, this is not a case of SPASS checking itself.

give the numbers of finite models and saturations found in the relevance checking done for EP proofs.

	EP	SPASS
Problems solved	48	83
Proofs checked	46	83
Obligations (generated)	592	19737
Obligations (discharged)	590	19737
Obligations / proof (avg.)	12.8	273.8
Obligations / proof		
1-10 theorems	35	52
10-100 theorems	10	13
100-1000 theorems	1	12
$>$ 1000 theorems	0	6
Discharge times / obligation		
0.0-0.1s	208	19737
0.1-0.2s	362	0
0.2-0.3s	17	0
$>$ 0.3s	3	0
Models found	361	-
Saturations found	0	-

Table 1: Proof Checking Results

The table shows that 46 or the 48 problems solved by EP were fully verified. Both failure cases were caused by Otter's inability to discharge obligations arising from steps in the FOF to CNF conversion. In particular, the obligations to verify the step that negates the conjecture, which entails proving the negation of the negation from the original, could not be discharged. All 83 of the SPASS proofs passed the verification checks chosen.

Most of the proofs require less than 10 obligations to be discharged, both for EP and SPASS. However, SPASS produces some very large proofs that consequently require a very large number of obligations to be discharged; the largest proof involved 3493 proof check obligations. This difference in distribution leads to a significant difference between the average numbers of obligations that had to be discharged per problem. This is despite the fact that the EP verifications included discharging obligations arising from the FOF to CNF conversions. At the same time, all of the SPASS obligations were discharged in almost no time. These figures indicate that SPASS proofs contain very many small, easily verified steps, while EP proofs have larger steps. There is some overhead starting Otter for each theorem obligation, and this dominates the wall clock time taken (i.e., the time the user has to wait for a proof to be verified). In this reality, it is preferable to have fewer but harder theorem obligations to discharge, as offered by EP.

Of the 590 obligations discharged for EP, 361 had the parents verified as satisfiable, confirming the relevance of the parents to the inferred clause. The remaining 229 theorems were not relevance checked because one of the parent clauses was derived from the negation of the conjecture.

6 Conclusions and Future Work

In this paper, we have described and evaluated a semantic derivation verification approach to proof checking. The evaluation, which is the main contribution of the paper, is based on 109 safety obligations arising in the certification of auto-generated aerospace code.

The results are encouraging. Our system is able to verify 129 out of the 131 proofs found by EP and SPASS, showing that the proof checking task is practically tractable. The vast majority of theorem obligations are discharged in less than 0.1 seconds. Most proofs are checked completely in less than 30 seconds wall clock time, although some of the longer proofs cannot be verified completely and even the partial checks can take more than five minutes.

There is still a lot of room for improvement. The verification of some trivial proof steps in the FOF-CNF conversion failed. The corresponding obligations were of the form $L \models \neg\neg L$, where L is very large. The trusted ATP (i.e., Otter) does not recognize this form and produces a very difficult CNF obligation. Using SPASS as the trusted ATP, however, solves this problem. Similarly, some forms of structural verification are very expensive, in particular for the large proofs found by SPASS. Moreover, the approach relies on the production ATP generating well documented proofs. Currently, only EP satisfies this criterion. SPASS proofs are missing the FOF-CNF conversion, and Vampire does not record the negation of the original goal, which makes its proofs uncheckable. Finally, we have evaluated our techniques only for ATPs based on the superposition calculus. Future work will thus be concerned with systems based on other calculi such as, for example, non-clausal resolution or model elimination.

Derivation verification does not provide absolute assurance. The biggest gap is the verification of Skolemization steps, which are only satisfiability preserving. While the full verification of such steps (and clausification in general) requires further research and experimentation, the partial verification provided here already gives some additional assurance. Other potential gaps are that the construction of the proof check obligations is wrong, and that the trusted ATP contains errors.

Ultimately, however, in order to convince users of the validity of the overall certification process, there needs to be some tracing between logical entities and the program being certified. In [DF05], we describe a browser which enables two-way linking between verification conditions and lines of the annotated program. We are also developing an extension to the VCG which adds "semantic markup" that can then be used to interpret the generated verification conditions. We would like to combine tracing, textual rendering, and proof checking into an integrated environment for certification.

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