an undecidable language

$H(i, j) \equiv M_i(j)$

is Recognizable (a.k.a. Recursively ~~Enum~~ Enumerable)

$U(i, j) \leftarrow$ Universal Turing machine

$U(i, j)$ When Accept provides a proof of j in the set
proof is the computation

$$\begin{cases} \longrightarrow \text{Reject (proof of } \notin) \\ \longrightarrow \perp \end{cases}$$

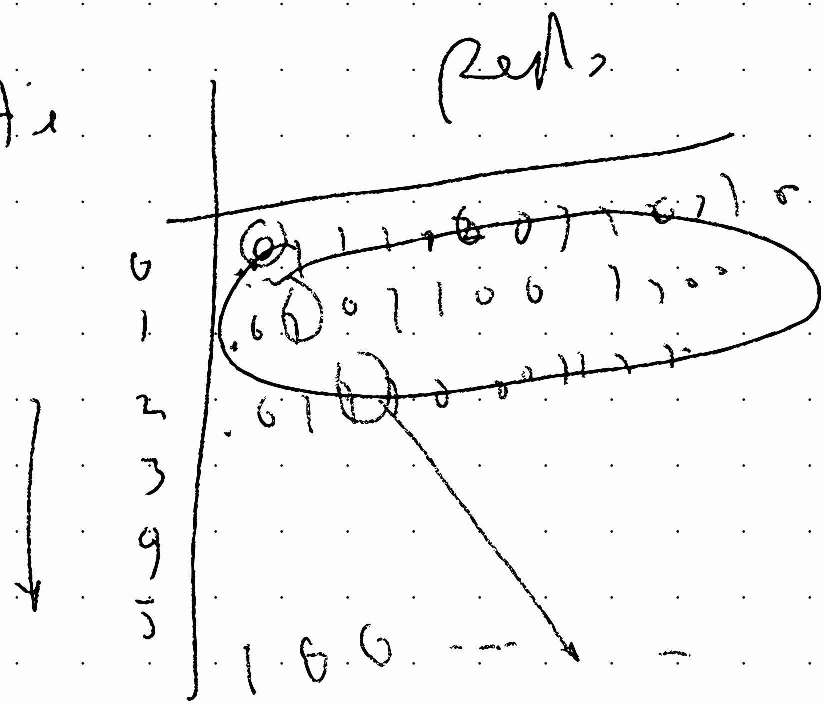
D ~~H~~	1	2	3	3	
M_1	T	~~T~~ F	T	T	~~T~~ F
M_2	T	~~T~~ F	T	~~T~~ F	~~T~~ F
M_3	T	~~T~~ F	T	~~T~~ F	~~T~~ F
	T	T	T	T	~~T~~ F
⋮					
M_j	F	T	F	F	---

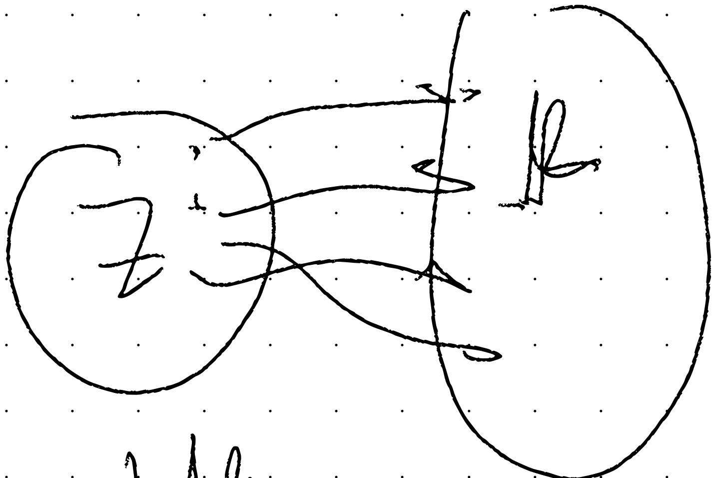
Diagonalization argument

how many real numbers are there?

$\mathbb{Z}_{\geq 0} \leftarrow$ infinite

$(0,1)$





Uncountable

Countable

$\mathbb{Q}$ rational numbers
countable

the set of TMs is a countable set

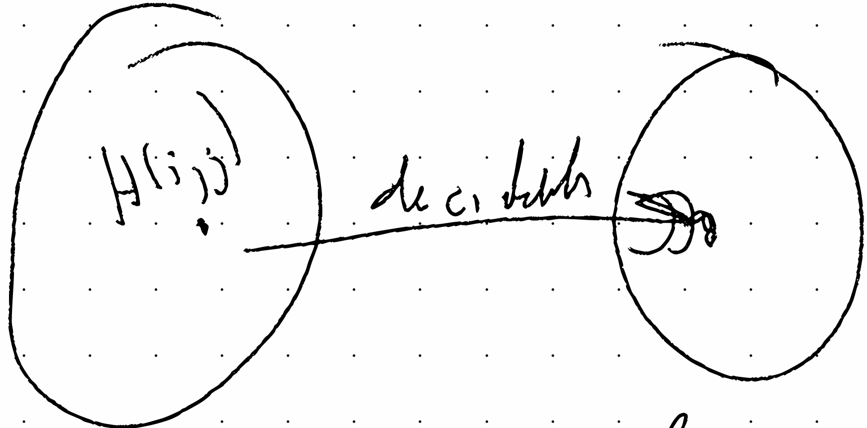
0 1 2 3 4 ...

0 1 1 0 1 0 0 0 1 1 1 ...

$\{\{1,2,3\}, \{7\}, \{6\}, ...\}$

~~the~~ set of subsets of the integers

uncountable



P

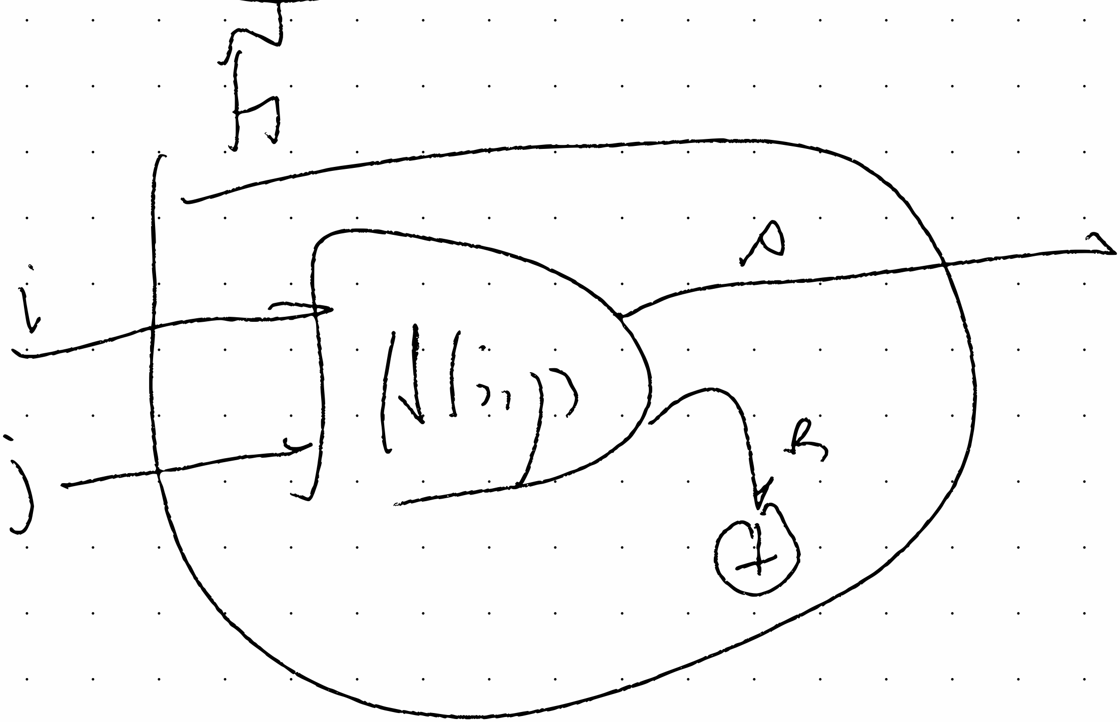
instance for problem

B

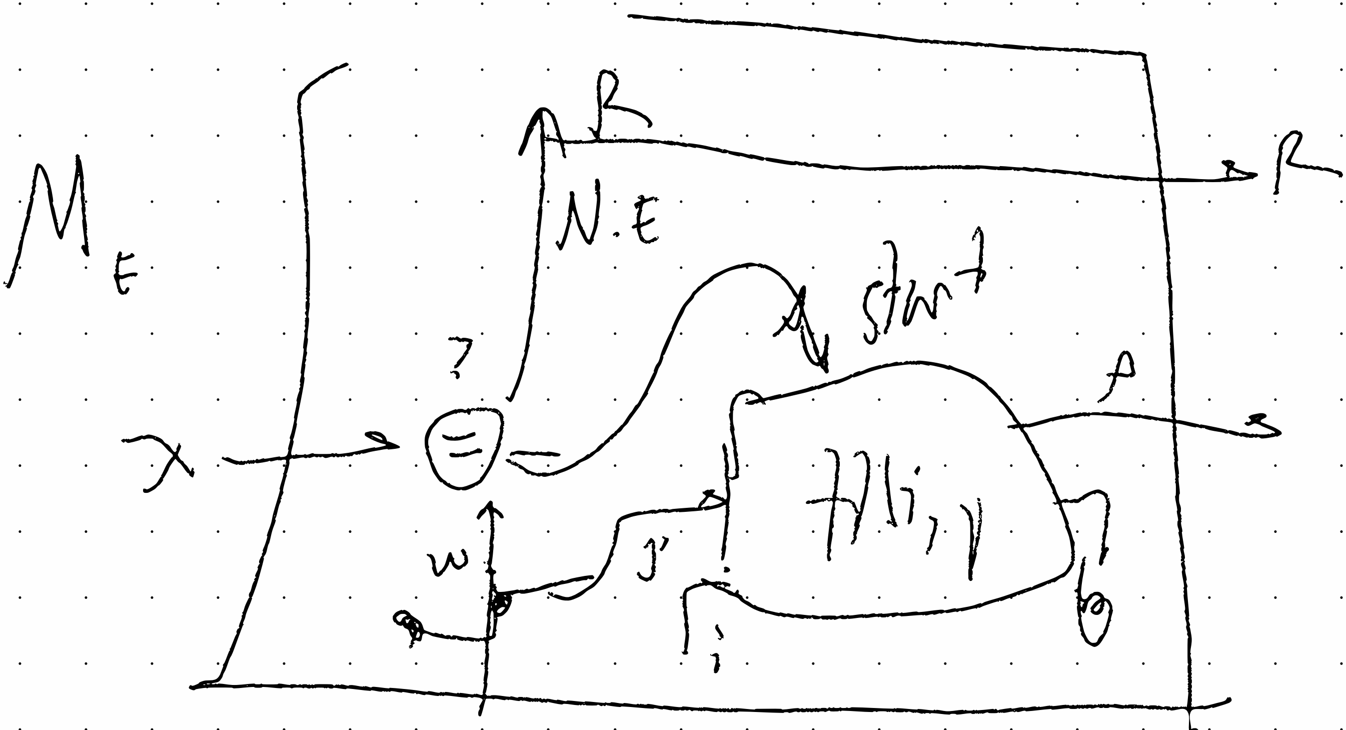
HALT(i,j)

$H(i,j) \in HALT(i,j)$

HALT is undecidable



$$L(M_E) \neq \emptyset \iff h(i, j) = A$$



if $H(A, i, j) = A \longrightarrow \Sigma^*$ regular

$\neq A \longrightarrow 0^k 1^k$ not regular