Midterm

Wednesday, 4 March 2020
9:10–10:00 AM

There are 5 problems each worth 6 points for a total of 30 points. Show all your work, partial credit will be awarded. Space is provided on the test for your work; if you use a blue book for additional workspace, sign it and return it with the test. No notes, no collaboration.

Name:

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1. Give an NFA that accepts exactly the strings over the alphabet \{0, 1\} such that the number of 01 substrings equals the number of 10 substrings. (Exactly means, those string and only those strings. The empty string happens to be such a string, by the way.)

Next, give a machine with the fewest number of states. Do not worry if you believe your first answer had the minimum number of states. This is just a problem to come back to later, to see if you can improve your otherwise correct solution.
2. Write a Regular Expression that expresses the same language as the following FSA.
3. Show that the language

\[ \{ a^i b^j c^k d^n \mid \text{where } i, j, k \geq 0 \text{ and } i + j + k = n \} \]

is not regular.
4. Give a Context Free Grammar for the language,

\[ \{ a^i b^j c^k \mid i = j \text{ or } i = k \} \]

Then show that the CFG is ambiguous by giving two parse trees in your grammar of the string \textit{aaabbbccc}. 
5. Give a Context Free Grammar for the Regular Expression:

\[ ab^*(a|b)(c(a|b))^* \]

Give a Regular Expression for the following Context Free Grammar, or give a proof or a concise logical argument why an equivalent Regular Expression does not exist,

\[
S \rightarrow AX \\
A \rightarrow aA | a \\
X \rightarrow \epsilon | abXc
\]