Basic introduction to logic

Semantic Web (CSC751)

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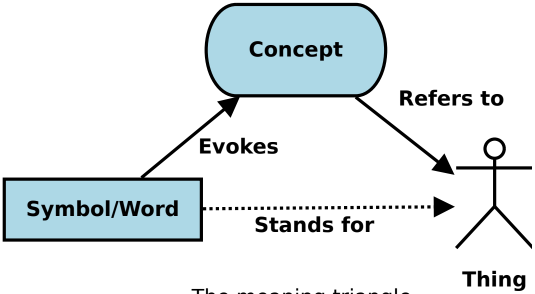
Outline

1. Announcements
2. Points to remember from previous discussion
3. Logic
4. Propositional logic
5. First order logic
6. Description logic



The meaning triangle

Announcements

Assignment

- Assignment #1
- Due on August 31^{st}; before the class starts

Reading

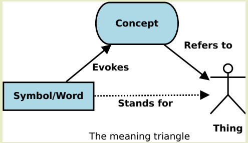
- (Mandatory) Read the papers [BLHL01, GS16, BHN16, Hit21]
- (Mandatory) Read Chapter 1 of the textbook
- (Mandatory) Appendix C.
- (Optional) Read Chapter 9
- (Optional) Appendix B.

Important points . . .

The basic idea of the Semantic Web

- is to provide a conceptual framework for
  1. **Build models** to capture the complexities of the world with simple methods through abstraction.
  2. **Compute meaningful conclusions** through a reasoning mechanism.
  3. **Communicate** unambiguous complex information though **ontologies**.

Building models [Mae02]



Basic ideas . . .

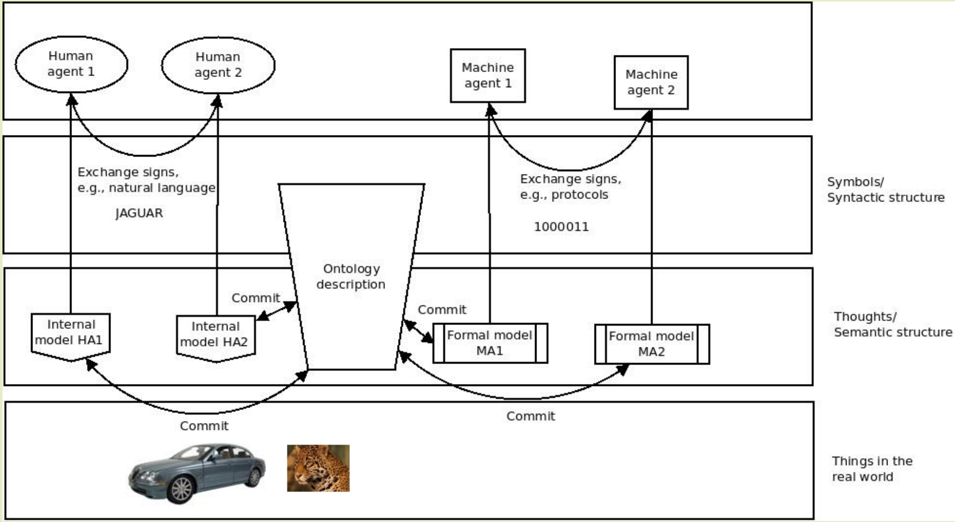
Compute meaningful conclusions[a]

[a] http://owl.man.ac.uk/2003/why/latest/

- $cat\_owner \equiv person \sqcap (\exists has\_pet.cat)$ (Cat owners have cat as pets)
- $has\_pet \sqsubseteq likes$ (has pet is a subproperty of likes, so anything that has a pet must like that pet)
- $cat\_liker \equiv person \sqcap (\exists likes.cat)$ (Cat owners must like a cat)
- Therefore, **Cat owners like cats.** (Justification: The subclass is inferred due to a subproperty assertion)

Basic ideas . . .

Communication [Mae02]



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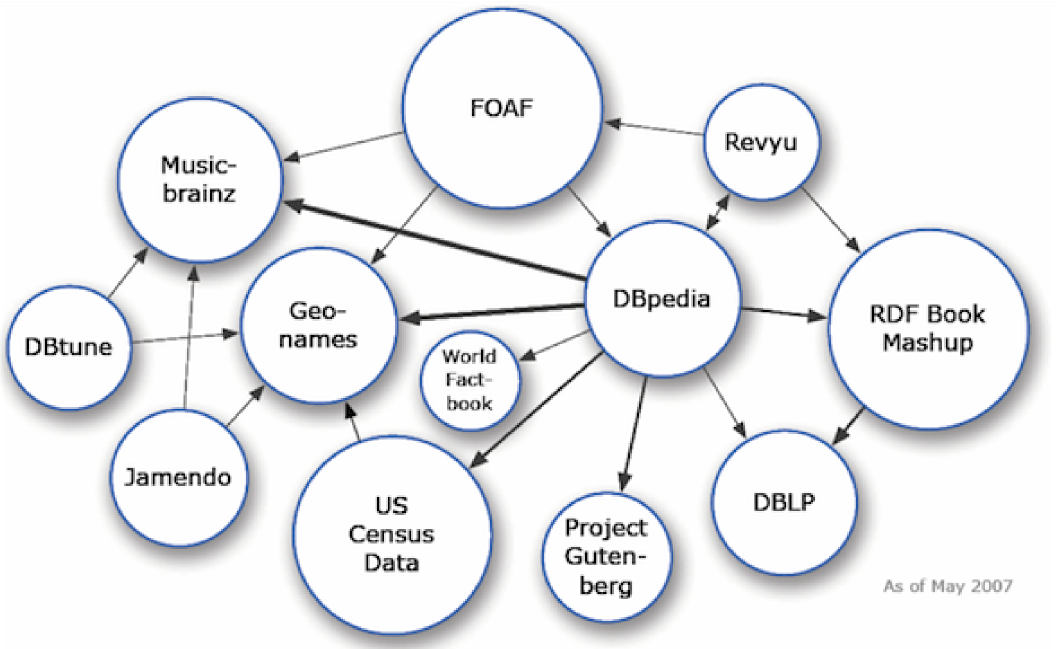

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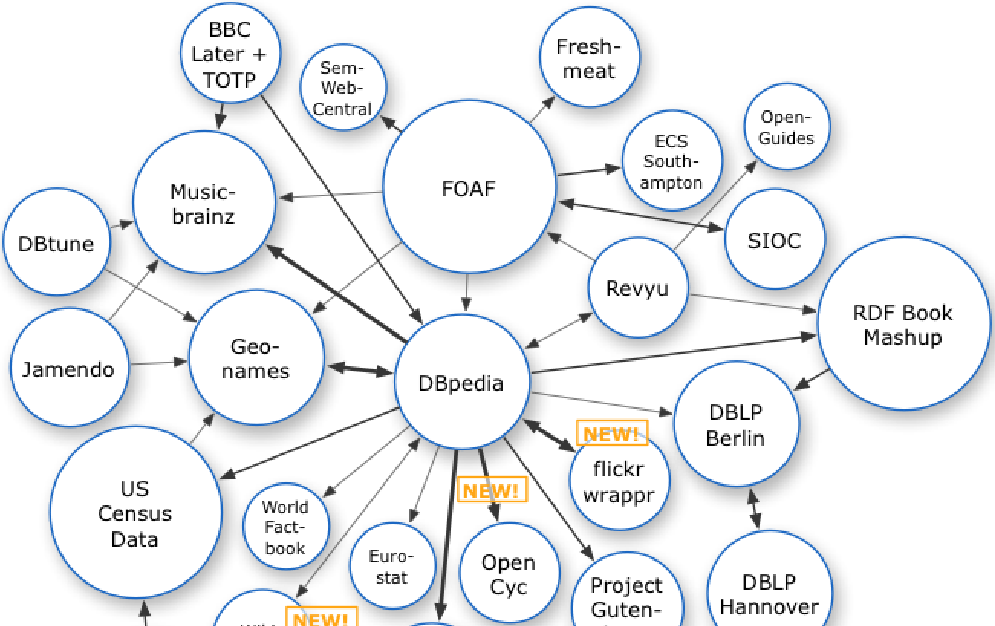

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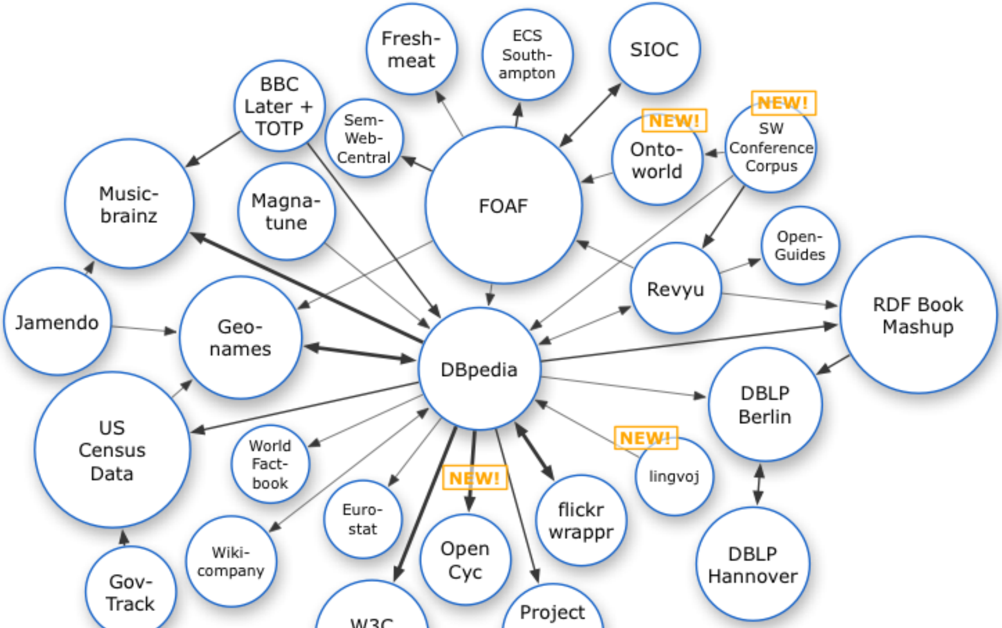

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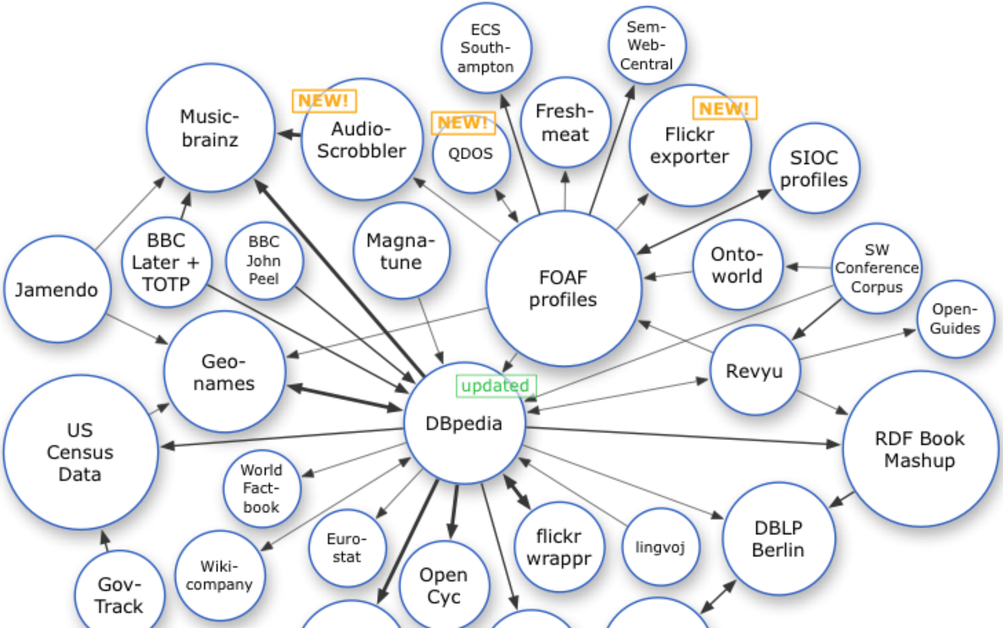
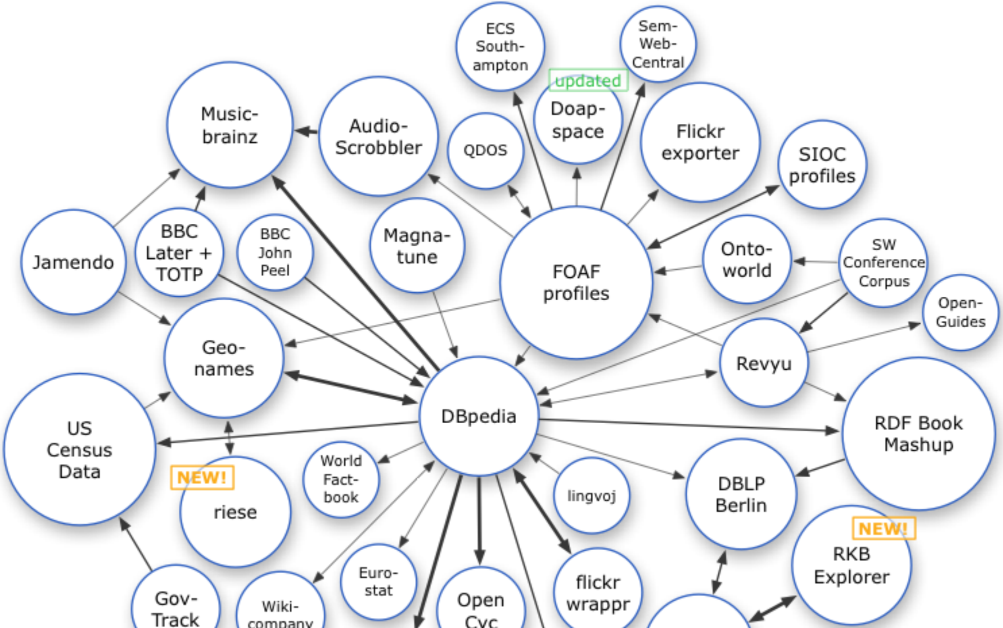


Linked Open Data, 03/2008



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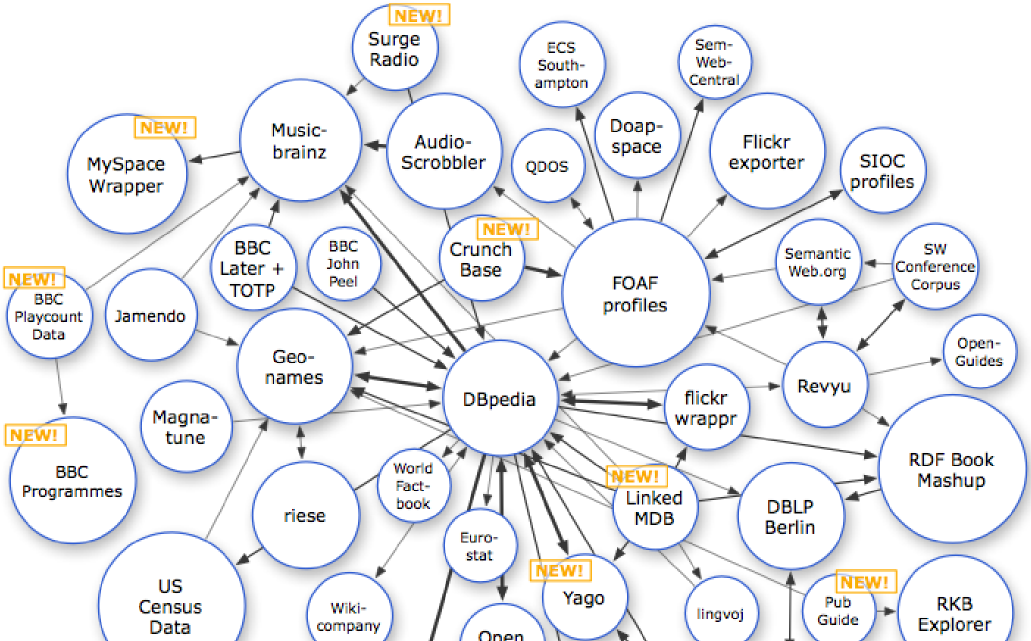

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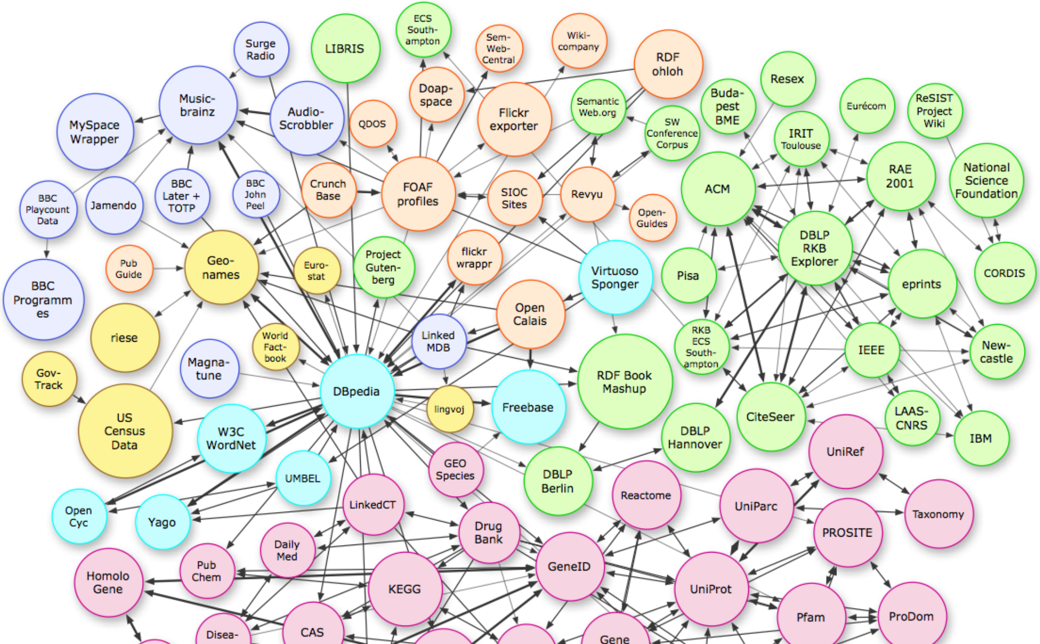

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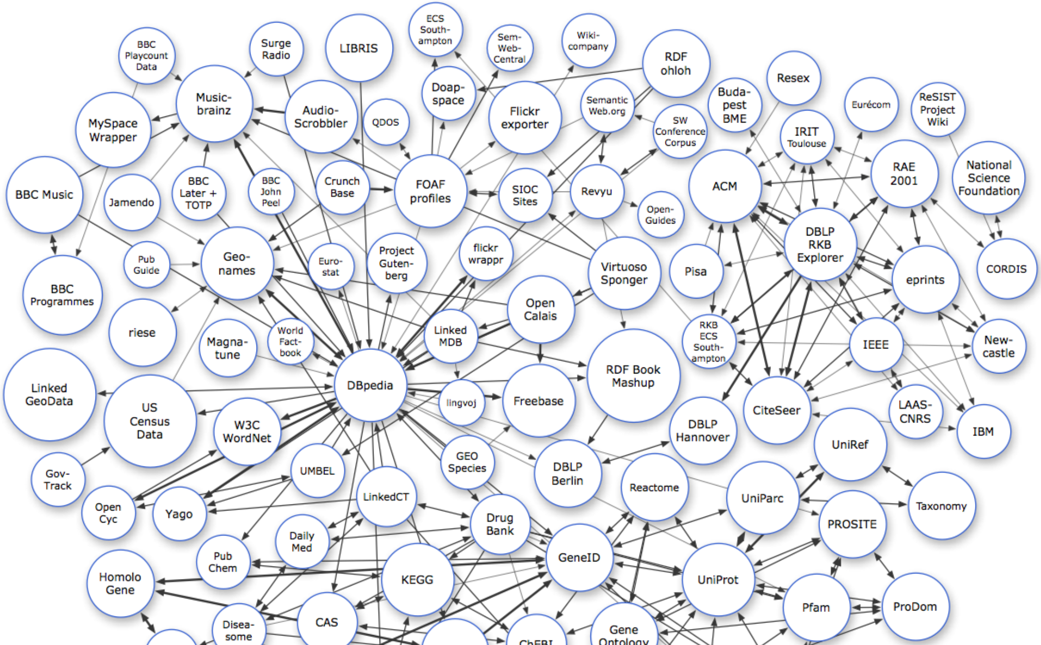

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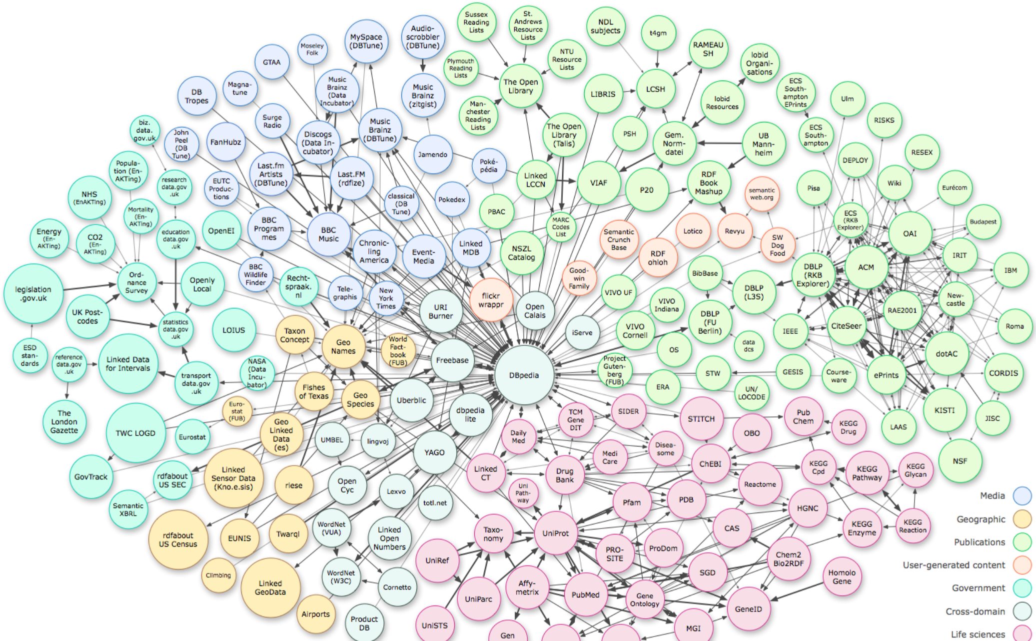

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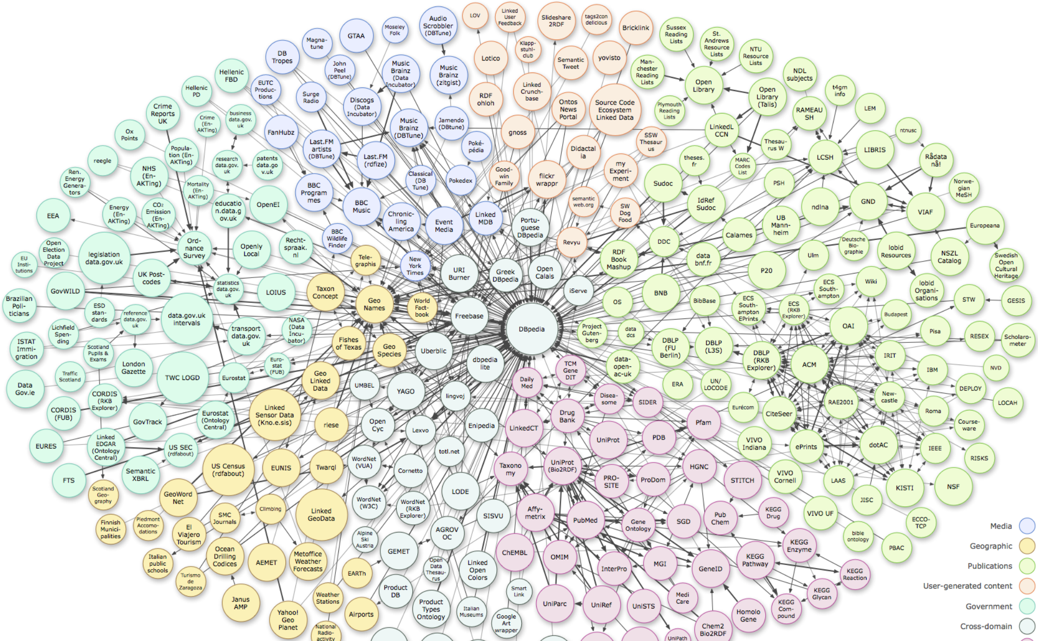

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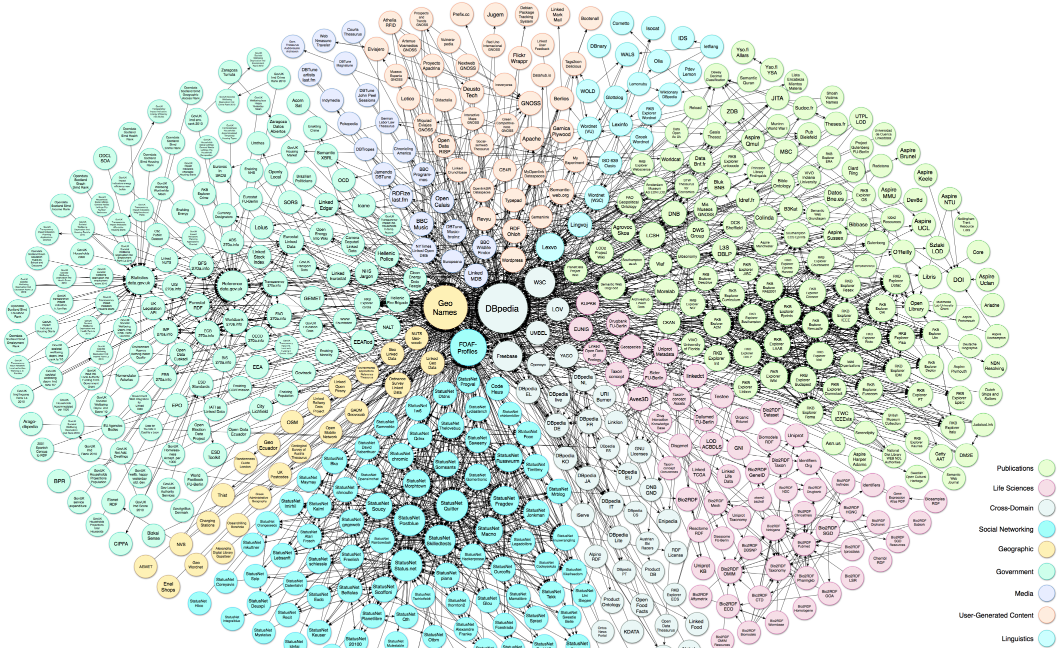
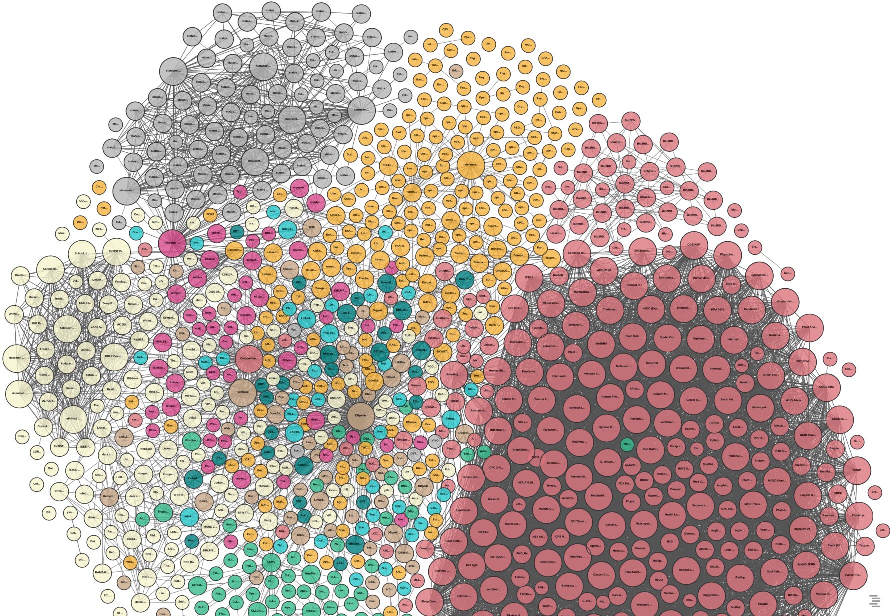


Linked Open Data, 02/2017

Legend

- Cross Domain
- Geography
- Government
- Life Sciences
- Linguistics
- Media
- Publications
- Social Networking
- User Generated
- Incoming Links
- Outgoing Links



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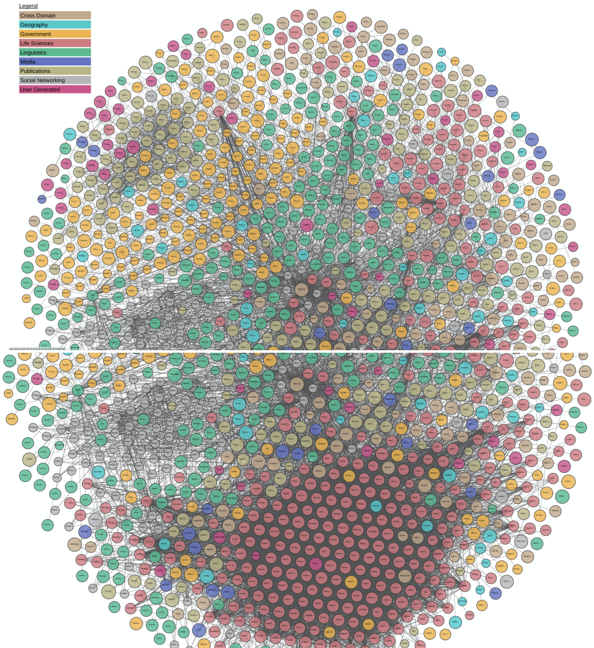

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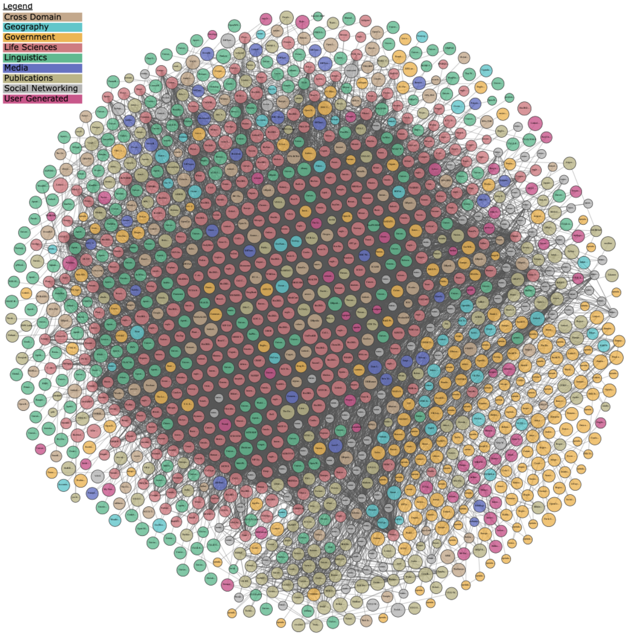

Theory Vs. Practice

"He who loves practice without theory is like the sailor who boards a ship without a rudder and compass and never knows where he may cast."
- Leonardo da Vinci

Obvious

"**Obvious** *is the most dangerous word in mathematics."*
- Eric Temple Bell

Logic: syntax, semantics, & models

. . . general properties

1. Logic provides an unambiguous and formal language to represent the world.
2. Every logic has a proper **syntax** to define sentences in the language, and
3. the **semantics** defines the **meaning** of these sentences.
4. A well define collection of sentences forms a **knowledge base (KB)**.

Meaning through models

Model

- We define the meaning of a sentence through **models**, i.e., the **truth-value** of a sentence in a **world**. *(Later we introduce the notion of an interpretation)*.
- If there is a sentence α, we say, **m** is a **model of** the sentence α, if α is **true** in **m**.
  e.g., $\alpha \equiv (x + 7 = 9)$; **true** in model $m = \{(x, 2)\}$
- We define the set of all models of α, $\mathbf{M}(\alpha)$.
  e.g., $\alpha \equiv (x + 7 = y)$; **true** in set $\mathbf{M}(\alpha) = \{\{(x, 0), (y, 7)\}, \{(x, -3), (y, 4)\}, \ldots\}$

Entailment (semantic relations between sentences)

We say a KB **entails**, i.e., one thing follows from another, a sentence α, $\mathbf{KB} \models \alpha$ if and only if $\mathbf{M}(\mathbf{KB}) \subseteq \mathbf{M}(\alpha)$.
e.g., If $\mathbf{KB} = \{x = 5\}$ then $\mathbf{KB} \models (x + 1) = 6$.

Propositional Logic: *Simplest logic ever ...*

Semantics

- Represent **facts** in the world.
  e.g., **birds are flying machines** is a fact, and the logic commits to either **true** or **false**.
- Such fact is given a proposition symbol, e.g., P, and models define the truth value of each symbol.
- A collection of facts create sentences through connectives. There are five connectives, $\neg$(*negation*), $\wedge$(*conjuction*), $\vee$(*disjunction*), $\Rightarrow$ (*implication*), $\Leftrightarrow$ (*biconditional*)

Illustration [RN09]

$\neg P$ true iff P is false in m
$P \wedge Q$ is true iff both P and Q are true in m
$P \vee Q$ is true iff either P or Q is true in m
$P \Rightarrow Q$ is true unless P is true and Q is false in m
$P \Leftrightarrow Q$ is true iff P and Q are both true or both false in m

Propositional Logic: *Simplest logic ever . . .*

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Truth table for connectives

$P, Q, \neg P, P \wedge Q, P \vee Q, P \Rightarrow Q, P \Leftrightarrow Q$

Logical equivalence [RN09]

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

and,
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$, i.e., $\mathbf{M}(\alpha) \equiv \mathbf{M}(\beta)$.

Inferences

Logical inference

- **Inference** is a procedure, i, that proves sentences from a KB.
- If a sentence α can be inferred from the KB using procedure i we say **KB** $\vdash_i \alpha$. Then, i is **sound** when **KB** $\vdash_i \alpha$ then **KB** $\models \alpha$, and **complete** when **KB** $\models \alpha$ then **KB** $\vdash_i \alpha$. *(Inference procedure derive a result in finitely many steps, $. \vdash .$)*
- A sound and a complete inference procedure correctly answers any question inferred from the KB. $O(2^n)$.

Illustration

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \vee Q$	**KB** $= P \wedge Q$
true	true	true	true	true	true
true	false	false	true	true	false
false	true	true	false	true	false
false	false	true	true	false	false

Inferences

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Illustration

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \vee Q$	**KB** $= P \wedge Q$
true	true	true	true	true	true
true	false	false	true	true	false
false	true	true	false	true	false
false	false	true	true	false	false

For theorem proving

Definitions

- A sentence is **valid** if it is **true** in **all models** (a.k.a. tautologies).
  e.g., $true, P \vee \neg P, (P \wedge (P \Rightarrow Q)) \Rightarrow Q$.
- **Deduction theorem:** For any sentence α and β, $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
- A sentence is **satisfiable** if it is **true** in **some** model.
  e.g., $P \vee Q$ is satisfiable in **m**$= \{\{P, true\}, \{Q, false\}\}$.
- A sentence is **unsatisfiable** if it has **no** models.
  e.g., $P \wedge \neg P$.
- So we do inference:
  KB $\models \alpha$ if and only if (**KB** $\wedge \neg\alpha$) is unsatisfiable.
- **Proof methods: Model checking** (DPLL), or applying a sequence of **inference rules**.

Inference

- **Modus Ponens**: $\frac{\alpha,\ \alpha\Rightarrow\beta}{\beta}$.
- **And-Elimination**: $\frac{\alpha\wedge\beta}{\alpha}$ or $\frac{\alpha\wedge\beta}{\beta}$
- **Bidirectional-Elimination**: $\frac{\alpha\Leftrightarrow\beta}{(\alpha\Rightarrow\beta)\wedge(\beta\Rightarrow\alpha)}$ and $\frac{(\alpha\Rightarrow\beta)\wedge(\beta\Rightarrow\alpha)}{\alpha\Leftrightarrow\beta}$.
- Sound and complete for propositional logic.

Resolution

- When sentences are in **Conjunctive Normal Form (CNF)**.
- Repeated application of resolution rule, $\frac{p_1\vee\ldots\vee p_k\vee\ldots\vee p_n, q_1\vee\ldots\vee\ldots q_n}{l_1\vee\ldots p_m\vee\ldots\vee l_{i-1}\vee l_{i+1}\vee\ldots\vee l_k\vee m_1\vee\ldots\vee m_{j-1}\vee m_{j+1}\vee\ldots\vee m_n}$.
  e.g., *Rain* $\Rightarrow$ *Wet*, *Wet* $\Rightarrow$ *Slippery*
  $\frac{\neg R\vee W, \neg W\vee S}{\neg R\vee S}$, i.e, *Rain* $\Rightarrow$ *Slippery*.

CNF steps; eliminate

1. $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
2. $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
3. Move $\neg$ inwards; $\neg(\neg\alpha) = \alpha$, and De Morgan $\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$, $\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$.
4. Apply distribution law; $(\ldots \vee \ldots) \wedge (\ldots \vee \ldots) \wedge \ldots$.

Logical reasoning

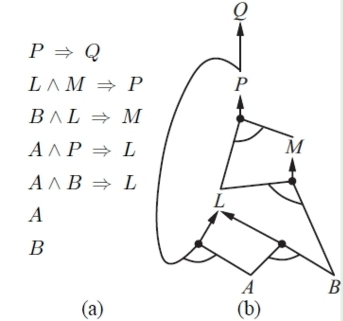
. . . with inference

- Complete.
- Need to search through an exponentially large space.
- 3-CNF, SAT is NP-complete (Theory of Computation). So there is unlikely a polynomial time algorithm.
- So we see for special sentences that are **easy** to prove.

Horn clauses

- There is **at most** one positive symbol.
- **(conjunction of symbol)** $\Rightarrow$ **symbol**.
  e.g., $(P \wedge Q \wedge R) \Rightarrow S$, i.e, $\neg(P \wedge Q \wedge R) \vee S$, $\neg P \vee \neg Q \vee \neg R \vee S$.
- Modus Ponens is complete for Horn clauses. Therefore, we use forward and backward chaining inference algorithms.

Forward chaining



Some details

- FC is data-driven and automatic. e.g., object recognition
- BC is goal-driven and used in problem-solving. e.g., Where is the Ungar building?

First order logic

. . . real world consists of

- **Objects**, e.g., Sam, Ubbo, . . .
- **Relations**, e.g., Ubbo isTheAdvisorOf Sam, . . .
- **Functions**, e.g., SamHead = head(Sam), . . .

Syntax

- **Constants**: e.g., Sam, Ubbo, UniversityOfMiami, . . .
- **Variables**: e.g., $x, y, z, \ldots$
- **Predicates**: e.g., isMemberOf, $>$, $=$, . . .
- **Functions**: e.g., SamHead = head(Sam), . . .
- **Connectives**: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- **Quantifiers**: $\exists, \forall$
- Always keep in mind that Constants, Predicates, and Functions are **just symbols**. There is **no** meaning on their own.

Semantics

Models in FOL

- Bit more complicated that propositional logic.
- Models contain
  1. a set of **objects**,
  2. a set of **relations** between objects (truth value mapping), and
  3. a set of **functions** that map objects to other objects.
- An **interpretation** provides the meaning of objects, relations and functions. It provides
  1. a mapping from constant symbols to model objects,
  2. a mapping from predicate symbols to model relations, and
  3. a mapping from function symbols to model functions.
- An interpretation is a model of a set of axioms if all the axioms are evaluated to true in the interpretation.
- Logical consequence: β is a logical consequence of α, $\alpha \models \beta$, if for all I with $I \models \alpha$, we also have $I \models \beta$. i.e., what implicit knowledge the KB entails.
- We refrain from using function as it is not used within the context of Semantic Web.

Semantics in more detail

Models one more time [Hor10]

- A model (a.k.a. an interpretation or a structure) is a pair $I = (D, .^I)$ with a set $D \neq \emptyset$ called the **domain** of I, and an interpretation function $.^I$,
  1. C^I is an element of D for C, a constant,
  2. v^I is an element of D for v, a variable,
  3. P^I is a subset of D^n for P, a predicate of arity n.

E.g., $D = \{a, b, c, d, e, f\}$, and

$\text{Felix}^I = a$

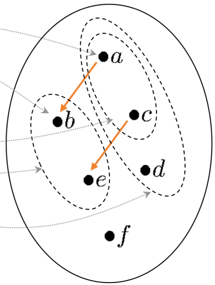
$\text{MyMat}^I = b$

$\text{Cat}^I = \{a, c\}$

$\text{Mat}^I = \{b, e\}$

$\text{Animal}^I = \{a, c, d\}$

$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$



Semantics

Evaluation [Hor10]

- Truth value of a given model $I = (D, .^I)$
  1. $P(t_1, \ldots, t_n)$ is **true** iff $< t_1^I, \ldots, t_n^I > \in P^I$,
  2. $P \wedge Q$ is **true** iff P is true and Q is true,
  3. $\neg P$ is **true** iff P is false.

E.g.,

Cat(Felix)	true
Cat(MyMat)	false
¬Mat(Felix)	true
sits-on(Felix, MyMat)	true
Mat(Felix) ∨ Cat(Felix)	true

$D = \{a, b, c, d, e, f\}$
$\text{Felix}^I = a$
$\text{MyMat}^I = b$
$\text{Cat}^I = \{a, c\}$
$\text{Mat}^I = \{b, e\}$
$\text{Animal}^I = \{a, c, d\}$
$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$

Semantics

Evaluation [Hor10]

- Truth value of a given model $I = (D, .^I)$
  1. $\exists x.P$ is **true** iff there exist an extended interpretation $.^{I'}$ such that $.^I$ and $.^{I'}$ differ w.r.t x, and P is true in $(D, .^{I'})$ **(Existential quantification)**.
  2. $\forall x.P$ is **true** iff there exist an extended interpretation $.^{I'}$ such that $.^I$ and $.^{I'}$ differ w.r.t x, and P is true in $(D, .^{I'})$ **(Universal quantification)**.

E.g.,

$\exists x.\mathrm{Cat}(x)$	true
$\forall x.\mathrm{Cat}(x)$	false
$\exists x.\mathrm{Cat}(x) \wedge \mathrm{Mat}(x)$	false
$\forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x)$	true
$\forall x.\mathrm{Cat}(x) \rightarrow (\exists y.\mathrm{Mat}(y) \wedge \text{sits-on}(x, y))$	true

$D = \{a, b, c, d, e, f\}$
$\mathrm{Felix}^I = a$
$\mathrm{MyMat}^I = b$
$\mathrm{Cat}^I = \{a, c\}$
$\mathrm{Mat}^I = \{b, e\}$
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$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$

Semantics

Evaluation [Hor10]

- Given a model I and a formula F, I is a model of F ($I \models F$), iff F is true in I.
- A formula F is **satisfiable** iff $\exists$ a model I such that $I \models F$.
- A formula entails another formula G ($F \models G$), iff every model of F is also a model of G. ($I \models F \Rightarrow I \models G$).

E.g.,

$M \models \exists x.\mathrm{Cat}(x)$

$M \not\models \forall x.\mathrm{Cat}(x)$

$M \not\models \exists x.\mathrm{Cat}(x) \wedge \mathrm{Mat}(x)$

$M \models \forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x)$

$M \models \forall x.\mathrm{Cat}(x) \rightarrow (\exists y.\mathrm{Mat}(y) \wedge \text{sits-on}(x, y))$

$D = \{a, b, c, d, e, f\}$

$\mathrm{Felix}^I = a$

$\mathrm{MyMat}^I = b$

$\mathrm{Cat}^I = \{a, c\}$

$\mathrm{Mat}^I = \{b, e\}$

$\mathrm{Animal}^I = \{a, c, d\}$

$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$

One more example [Hor10]

E.g.,

- ✓ $\text{Cat(Felix)} \models \exists x.\text{Cat}(x)$ $\quad (\text{Cat(Felix)} \wedge \neg \exists x.\text{Cat}(x)$ is not satisfiable)
- ✓ $(\forall x.\text{Cat}(x) \rightarrow \text{Animal}(x)) \wedge \text{Cat(Felix)} \models \text{Animal(Felix)}$
- ✓ $(\forall x.\text{Cat}(x) \rightarrow \text{Animal}(x)) \wedge \neg\text{Animal(Felix)} \models \neg\text{Cat(Felix)}$
- ✗ $\text{Cat(Felix)} \models \forall x.\text{Cat}(x)$
- ✗ $\text{sits-on(Felix, Mat1)} \wedge \text{sits-on(Tiddles, Mat2)} \models \neg\text{sits-on(Felix, Mat2)}$
- ✗ $\text{sits-on(Felix, Mat1)} \wedge \text{sits-on(Tiddles, Mat1)} \models \exists^{\geq 2} x.\text{sits-on}(x, \text{Mat1})$

Note that $\exists^{\geq n}, \exists^{\leq n}$ are called counting quantifiers.

$\exists^{\geq 3} x.Cat(x) \equiv \exists x, y, z.Cat(x) \wedge Cat(y) \wedge Cat(z) \wedge x \neq y \wedge x \neq z \wedge y \neq z$

$\exists^{\leq 2} x.Cat(x) \equiv \forall x, y, z.Cat(x) \wedge Cat(y) \wedge Cat(z) \Rightarrow x = y \vee x = z \vee y = z$.

... just one more example

$\forall x(Exam(x) \Rightarrow \forall y(hasExaminer(x, y) \Rightarrow Professor(y)))$

Examiners of an exam must be professors.

If $Exam^I = D \in \mathbb{N}$, $hasExaminer^I = \{(n, m) | n \leq m\}$, $Professor^I = \{n + n | n \in D\}$, then under this interpretation, every non-negative integer n we have that $m \geq n$ is an even number.

Correct interpretation are all models of the formula.

Why do we need Description Logic? *Thank you for asking*

Decidability

- We know that a deduction calculus (a.k.a. inference procedure) is **sound** if $T \vdash F$ implies $T \models F$. It is **complete** if $T \models F$ implies $T \vdash F$.
- FOL is **semi-decidable**, i.e., when $T \models F$ implies $T \vdash F$. This means that we can have a concrete algorithm that terminates and returns the correct answer. However, when $T \not\models F$ then the deduction algorithm is generally unbounded.
- **Decision procedure**: a sound and a complete algorithm that is guaranteed to terminate on all inputs. Therefore,
- Description logic deals only with decidable fragments of FOL.
- We say DL is in the family of **C2**, i.e., FOL with **two** variables and counting quantifiers.
- So which decidable fragments do I have to work with?

DL syntax

Signature

DL	FOL	Example	Note
Concept (or Class) names	unary predicates	Cat, Animal, Person	Equivalent to FOL unary predicates
Property names	binary predicates	isAdvisorOf, loves	Equivalent to FOL binary predicates
Individual names	constants	Sam, Ubbo, Felix,	Equivalent to FOL constants

DL syntax

Operators

- Conjunction $\sqcap$; which is interpreted as the set intersection.
- Negation $\neg$; which is interpreted as the set complement.
- Disjunction $\sqcup$; which is interpreted as the set union.
- Universal restriction $\forall R.C$,
- Existential restriction $\exists R.C$,
- Number restriction $\geq nR$, $\leq nR$, and, $= nR$.
- Subsumption $\sqsubseteq$; which is interpreted as the material implication.
- Equivalence $\equiv$; which is interpreted as the equivalence.
- Role inclusions $\circ$.

{T/A}Box

- Terminological axioms (**TBox**) introduce names for complex descriptions.
- Assertional formalisms (**ABox**) states properties of individuals. This is also know as grounded facts.

DL syntax

Special concepts

- $\top$; Thing, the most general concept.
- $\bot$; Nothing, inconsistent concept.

Illustrations (one/two free variables)

DL	**FOL**
Doctor $\sqcup$ Lawyer	Doctor$(x) \vee$ Lawyer(x)
Rich $\sqcap$ Happy	Rich$(x) \wedge$ Happy(x)
Cat $\sqcap$ $\exists$sitsOn.Mat	$\exists y$(Cat$(x) \wedge$ sitsOn(x, y))
loves^{-}	loves(y, x)
hasParent $\circ$ hasBrother	$\exists z$ (hasParent$(x, z) \wedge$ hasBrother(z, y))

DL syntax

Illustrations: TBox [Hor10]

DL:

$\mathrm{Rich} \sqsubseteq \neg\mathrm{Poor}$	(concept inclusion)
$\mathrm{Cat} \sqcap \exists\text{sits-on}.\mathrm{Mat} \sqsubseteq \mathrm{Happy}$	(concept inclusion)
$\mathrm{BlackCat} \equiv \mathrm{Cat} \sqcap \exists\mathrm{hasColour}.\mathrm{Black}$	(concept equivalence)
$\text{sits-on} \sqsubseteq \mathrm{touches}$	(role inclusion)
$\mathrm{Trans}(\text{part-of})$	(transitivity)

Equivalent FOL:

$$\forall x.(\mathrm{Rich}(x) \rightarrow \neg\mathrm{Poor}(x))$$

$$\forall x.(\mathrm{Cat}(x) \wedge \exists y.(\text{sits-on}(x,y) \wedge \mathrm{Mat}(y)) \rightarrow \mathrm{Happy}(x))$$

$$\forall x.(\mathrm{BlackCat}(x) \leftrightarrow (\mathrm{Cat}(x) \wedge \exists y.(\mathrm{hasColour}(x,y) \wedge \mathrm{Black}(y)))$$

$$\forall x,y.(\text{sits-on}(x,y) \rightarrow \mathrm{touches}(x,y))$$

$$\forall x,y,z.((\text{sits-on}(x,y) \wedge \text{sits-on}(y,z)) \rightarrow \text{sits-on}(x,z))$$

DL syntax

Illustrations: TBox

- Woman $\sqsubseteq$ Person ; ?
- Person $\equiv$ HumanBeing; ?
- hasWife $\sqsubseteq$ hasSpouse; $\forall x, y(\text{hasWife}(x, y) \Rightarrow \text{hasSpouse}(x, y)$
- hasSpouse $\equiv$ marriedWith; ?
- hasParent $\circ$ hasBrother $\sqsubseteq$ hasUncle; $\forall x, y(\exists z((\text{hasParent}(x, z) \wedge (\text{hasBrother}(z, y))) \Rightarrow \text{hasUncle}(x, y)\ ))$

Illustrations: ABox [Hor10]

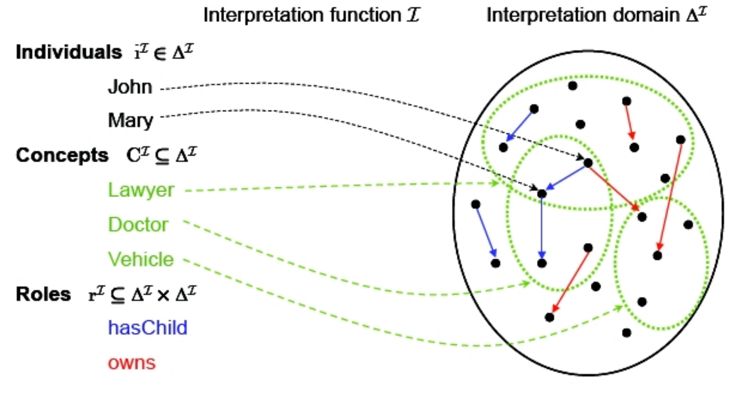
BlackCat(Felix)	(concept assertion)
Mat(Mat1)	(concept assertion)
Sits-on(Felix,Mat1)	(role assertion)

- Person(mary); Person(john)
- hasChild(john, mary);

DL semantics

Semantics [Hor10]

- Directly using FOL model theoretic view.



DL semantics [Hor10]

Semantics

- The interpretation *I* directly extends to **concept expressions**

$$
\begin{aligned}
&(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
&(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
&(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
&\{x\}^{\mathcal{I}} = \{x^{\mathcal{I}}\} \\
&(\exists R.C)^{\mathcal{I}} = \{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \\
&(\forall R.C)^{\mathcal{I}} = \{x \mid \forall y.(x, y) \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\} \\
&(\leqslant n R)^{\mathcal{I}} = \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \leqslant n\} \\
&(\geqslant n R)^{\mathcal{I}} = \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \geqslant n\}
\end{aligned}
$$

DL semantics

Semantics

- Given an interpretation $I = (D, .^I)$
  - $I \models C \sqsubseteq D$ iff $C^I \subseteq D^I$,
  - $I \models C \equiv D$ iff $C^I = D^I$,
  - $I \models C(a)$ iff $a^I \in C^I$,
  - $I \models R(a, b)$ iff $< a^I, b^I > \in R^I$,
  - $I \models < TBox, ABox >$ iff for every axiom $a_x \in TBox \cup ABox, I \models a_x$, and
  - a DL knowledge base is TBox plus ABox, which is written as $K =< TBox, ABox >$.
- We say K is satisfiable iff $\exists$ an interpretation (or model) I s.t $I \models K$,
- A concept C is satisfiable w.r.t. K, iff $\exists$ $I = (D, .^I)$ s.t $I \models K$ and $C^I \neq \emptyset$.
- K entails an axiom, $K \models a_x$ iff for every model I of K, $I \models a_x$, i.e., $I \models K$ implies $I \models a_x$.

DL expressivity: Many different DLs

ALC: smallest possibly closed DL

- TBox expressions:
  - Subclass relationship, $\sqsubseteq$, and equivalence, $\equiv$.
  - Conjunction, $\sqcup$, disjunction, $\sqcap$, and negation $\neg$.
  - Property restriction, $\forall$, and $\exists$.
  - Also, $\top$, and $\bot$.

    e.g., ProudParent $\equiv$ Person $\sqcap$ $\forall$ hasChild.(Doctor $\sqcup$ $\exists$ hasChild.Doctor)

DL expressivity

Other extensions

- ALC + role chains = SR. Role chains include
  - hasParent $\circ$ hasBrother $\sqsubseteq$ hasUncle (also include top property and bottom property)
  - Transitivity; (hasAncestor $\circ$ hasAncestor $\sqsubseteq$ hasAncestor)
  - Role hierarchies; (hasFather $\sqsubseteq$ hasParent)
- O - nominals (closed classes) (MyBirthdayGuests = {bill,john,mary})
- I - inverse roles; (hasParent $\equiv$ hasChild^{-})
- Q - quantified cardinality restrictions (Car $\sqsubseteq$ =4hasTyre.T)

Complexity [HKR09]

- ACL; ExpTime.
- SHIQ, SHOQ, and SHIO; ExpTime.
- SHOIQ; NExpTime.
- SROIQ; N2ExpTime.
- SROIQ^{D}; We learn about this family next week, when we talk about **ontologies**.

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Abraham Bernstein, James Hendler, and Natalya Noy.
A new look at the Semantic Web.
Communications of the ACM, 59(9):35–37, 2016.

Tim Berners-Lee, James Hendler, and Ora Lassila.
The Semantic Web.
Scientific American, 284(5):34–43, May 2001.

Birte Glimm and Heiner Stuckenschmidt.
15 Years of Semantic Web: An Incomplete Survey.
KI - Künstliche Intelligenz, 30(2):117–130, 2016.

Pascal Hitzler.
A Review of the Semantic Web.
Communications of the ACM, 64(2):76–83, February 2021.

Pascal Hitzler, Markus Krötzsch, and Sebastian Rudolph.
Foundations of Semantic Web Technologies.
Chapman & Hall/CRC, 2009.

Ian Horrocks.
Description Logic: A formal foundation for languages and tools. Tutorial at the Semantic Technology Conference (SemTech). San Francisco, California, USA.

http://www.cs.ox.ac.uk/people/ian.horrocks/Seminars/seminars.html#tutorials, 2010.

Alexander D. Maedche.
Ontology Learning for the Semantic Web.
Kluwer Academic Publishers, Norwell, MA, USA, 2002.

S. J. Russell and P. Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2009.