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Iterative Algorithms for Performance Evaluation of Wireless Networks with Guard Channels

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Analytical and modeling work performed since the late eighties have led to nonlinear equations that relate important parameters used for performance evaluation of wireless networks. These nonlinear equations have no closed-form solution and iterative algorithms are used to find numerical solutions. However, iterative algorithms that are not designed to detect and overcome nonconvergence situations may fail to converge. We present an iterative algorithm that numerically solves six dependent nonlinear equations. The algorithm always converges and obtains values of *blocking probability,* p_b, and *forced termination probability,* p_{ft}, at any desired level of accuracy. We then used this algorithm to numerically show that for a given pair of values of p_b and p_{ft}, there is an optimal number of guard channels that supports a maximal new-call arrival rate.

KEY WORDS: Cellular systems; guard channels; iterative algorithms; Markov process; performance evaluation.

1. INTRODUCTION

In a wireless network, the service area is divided into cells. When a mobile terminal (MT) requests service, it may either be assigned a channel in its resident cell or denied service. This denial of service is known as *call blocking,* and its probability, p_b, as *call blocking probability.* A currently serving MT in a wireless network may move from one cell to another; the continuity of service to the MT in the new cell requires a successful *handoff* from the previous cell to the new cell. A handoff is successful if a channel is available and allocated for the MT's use. The probability of a handoff failure is called *handoff failure probability,* denoted here as p_{hf}. During the life of a call, an MT may cross several cell boundaries and hence may require several successful handoffs. Failure to get a successful handoff at any cell crossing forces the network to discontinue service to the MT. This is known as *forced termination* of the call; the probability p_{ft} of such an event occurring is known as *forced termination probability.*

Two most commonly used quality-of-service indicator parameters, p_b and P_{ft}, are estimated by (1) analytical and/or (2) simulation modeling from four basic parameters—*new call arrival rate, average call holding time, average cell dwell time,* and *number of channels in each cell.* Simulation models, being conceptually easy to understand and flexible to model real-world situations, are very widely used during the system design phase. However, development of any realistic simulation model involves significant programming overhead. Moreover, a simulation run takes relatively long computing time. In addition, to eliminate any possible bias of a starting state of the system being simulated, several simulation runs must be used to confirm the consistency of the obtained parameters. Nonetheless, simulation models are the only way to estimate a system's parameters if no analytical model is available.

Analytical modeling work performed since the late eighties have led to nonlinear equations that relate these parameters, with the following basic assumptions [1,11,5,6,3,2,12,9]. New call arrivals form a Poisson process with arrival rate λ_0. The holding time of calls is exponentially distributed with a mean $1/\mu$ seconds. Aver-

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age dwell time of calls—the duration of time a call stays in a cell—is exponentially distributed with mean $1/\eta$ seconds. The arrival process of handoff calls a Poisson [5], but the arrival rate of handoff calls (depends on *and*) is determined by other system parameters that are related by a set of nonlinear equations. The time interval between two consecutive phone calls to a mobile is sufficiently larger than the call holding time such that the busy-line effect does not occur.

In Section 2, we briefly review the equations that have been obtained assuming a Markov process. Because the equations are nonlinear and have no closed-form solution, iterative algorithms are necessary to find numerical solutions. However, iterative algorithms may fail to converge. To the best of our knowledge, there is no published algorithm to deal with the nonconvergence problem for this set of equations. We present a basic iterative algorithm in Section 3. The algorithm always converges and obtains values of p_b and p_{hf} at any desired level of accuracy.[2]

It is customary and essential to verify iterative algorithms by comparing their solutions with simulation or real data. We verify our iterative algorithm by simulation data. We use a simulation model, which is presented Section 4.

Another QoS (quality of service) parameter, p_{ft}, is computed using the value of p_{hf}. Increasing demands for mobile services, limited availability of bandwidth, and the smaller cell size to support more simultaneous calls can increase the p_{ft} of calls. Two handoff prioritization schemes, (1) advanced request for a channel—*prerequest scheme* [4–6], and (2) reserving a number of channels for only handoff calls—*guard channel scheme* [3,4,12,10], are used to reduce p_{ft}. Here we focus on the guard channel scheme. The algorithm presented in [9] computes the minimum number of channels (including the guard channels) required in each cell when the arrival rates of new and handoff calls, and their respective blocking probabilities (p_b and p_{hf}), are known. The algorithm reported in [10] makes assumptions identical to those in [9]; however, utilizing the newly proposed concept of the *fractional guard channel* policy, instead of the well-known *(integral)* guard channel policy, it shows a lower probability of new call blocking. Here we address the problem of computing the optimal number of guard channels when the number of available channels in a cell is a known constant but the handoff call arrival rate is not a constant and changes as the number of guard channels is changed. In particular, for a given QoS, that is, a pair of values of p_b and p_{ft}, we numerically show that there is an optimal number of guard channels that supports a maximal new call arrival rate, if no *direct assumption* about the arrival rate of handoff calls is made. The algorithm to compute the optimal number of guard channels is presented in Section 5. Finally, in Section 6 we show the agreement of numeric values obtained from our algorithm with those from simulations.

[2] The algorithms presented in [9] and [10] assume that arrival rates of handoff calls is known. We make no such assumptions.

2. MARKOV MODEL

It is assumed that g of the s channels in a cell are reserved for handoff arrival calls only, and the remaining $n = s - g$ channels can be used by both new and handoff calls. In the event of a new call arrival, it is accepted if the number of busy channels in the target cell at that time is less than n. Otherwise, the call is blocked. If there is a handoff arrival, it is blocked only if there is no free channel in the target cell, that is, all channels in the target cell are busy. This system can be modeled as a Markov process with $s + 1$ states (see Fig. 1 for a state diagram) with the' steady-state probabilities P_j, $0 \leq j \leq s$. For brevity, after defining $\rho_0 = \lambda_0/\mu$, $\rho = (\lambda_0 + \lambda_h)/(\mu + \eta)$, $\rho_h = \lambda_h/(\mu + \eta)$, steady-state probabilities P_j can be calculated from the following three equations (see [6,2] for details):

$$P_j = \frac{\rho^j}{j!} P_0, \qquad 0 < j \leq n \tag{1}$$

$$P_j = \frac{\rho^n \rho_h^{j-n}}{j!} P_0, \qquad n < j \leq s \tag{2}$$

$$P_0 = \left[\sum_{j=0}^{n} \frac{\rho^j}{j!} + \sum_{j=n+1}^{s} \frac{\rho^n \rho_h^{j-n}}{j!} \right]^{-1} \tag{3}$$

The blocking probability, p_b, of a new call, and the handoff failure probability, p_{hf}, of an ongoing call, are given by

$$p_b = \sum_{j=n}^{s} P_j \tag{4}$$

$$p_{hf} = P_s \tag{5}$$

The handoff call arrival rate, λ_h, because of user mobility is given by [5]

$$\lambda_h = \frac{\eta(1 + p_b)}{\mu + \eta p_{hf}} \lambda_0 \tag{6}$$

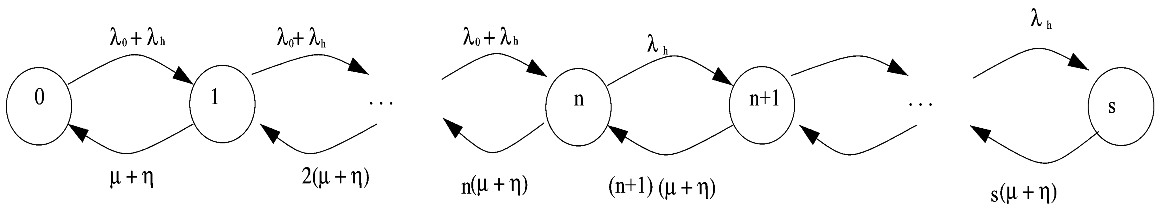

Fig. 1. The state diagram for the guard channel scheme.

Finally, the probability of forced termination because of a handoff failure is given by

$$p_{ft} = 1/\left(1 + \frac{\mu}{\eta p_{hf}}\right) \tag{7}$$

3. BASIC ITERATIVE ALGORITHM

The first six nonlinear independent equations described in the previous section have no closed-form solution. Therefore, iterative algorithms are necessary for finding numerical solutions. However, iterative algorithms may fail to converge. To the best of our knowledge, there is no published algorithm to deal with the nonconvergence problem for this set of equations. Here we present an algorithm that attempts to find numeric solutions iteratively, while keeping a count on iterations for detecting possible nonconvergence. If nonconvergence is suspected, a linear search is performed to obtain a solution with the desired level of accuracy. The complexity of the linear search is inversely proportional to the desired level of accuracy.

In any numerical solution of a set of nonlinear equations, another issue that needs careful attention is the relative accuracy of the computation. Because a mobile system with "good" QoS ought to maintain a very small value of p_{ft}, and possibly a small value of p_b, we define a relative accuracy coefficient—*rel-accur-coef*—as a minimum of desired p_{ft}, and p_b. The difference between two consecutive values of p_b is divided by *rel-accur-coef* to obtain error. The iterative process stops when the calculated error falls below the desired accuracy or a nonconvergence situation is suspected; in the latter case, a linear search is performed. The pseudo code of the algorithm is presented next.

Procedure *Find_Probabilities* (Input: ρ_0, *g;* Output: p_b, p_{ft})
  begin
    set $\lambda_0 = \rho_0\mu$;
    set $\lambda_h = 0.1 \times \lambda_0$; //a nonzero value to start computation
    set *error* $= 10 \times$ *accuracy;* // a number bigger than the accuracy
    set $n = s - g$, *count* $= 0$;
    while ((*error* $>$ *accuracy*) or (*count* $< 1/$*accuracy*)) **do**
    begin // computing values
      compute P_0 using equation (3);
      compute P_n using equation (1);
      compute P_j, $n < j \leq s$ using equation (2);
      compute p_b using equation (4);
      compute p_{hf} using equation (5);
      compute λ_h using equation (6);
      set *error* $= |$*previous_* $p_b - p_b|$/*rel-accur-coef;*
      increment *count* by 1;
    end;
    compute p_{ft} using equation (7);
    if (*error* $>$ *accuracy*) **then** // value of λ_h does not converge
    begin
      set λ_h-lower-end $=$ min(λ_h,previous-λ_h);
      set λ_h-upper-end $=$ max(λ_h,previous-λ_h);
      set $\delta\lambda_h$ $=$ (λ_h-upper-end $-$ λ_h-lower-end) *accuracy;*
      linear-search for best λ_h; // between lower and upper end of λ_h with increment of $\delta\lambda_h$
    end;
  end.

The data obtained from the iterative algorithm presented here are compared with that obtained from simulations reported in the next section.

4. SIMULATION MODEL

For the simulation, we considered hexagonal cells with wraparound topology, because it eliminates the boundary effect, keeping exactly six neighbors for each cell [2]. The 49 white cells, shown in Fig. 2 are part of the simulation model, and the shaded ones are wraparound neighbors of the boundary cells. All cells received 20 channels. The mobility of MTs is modeled

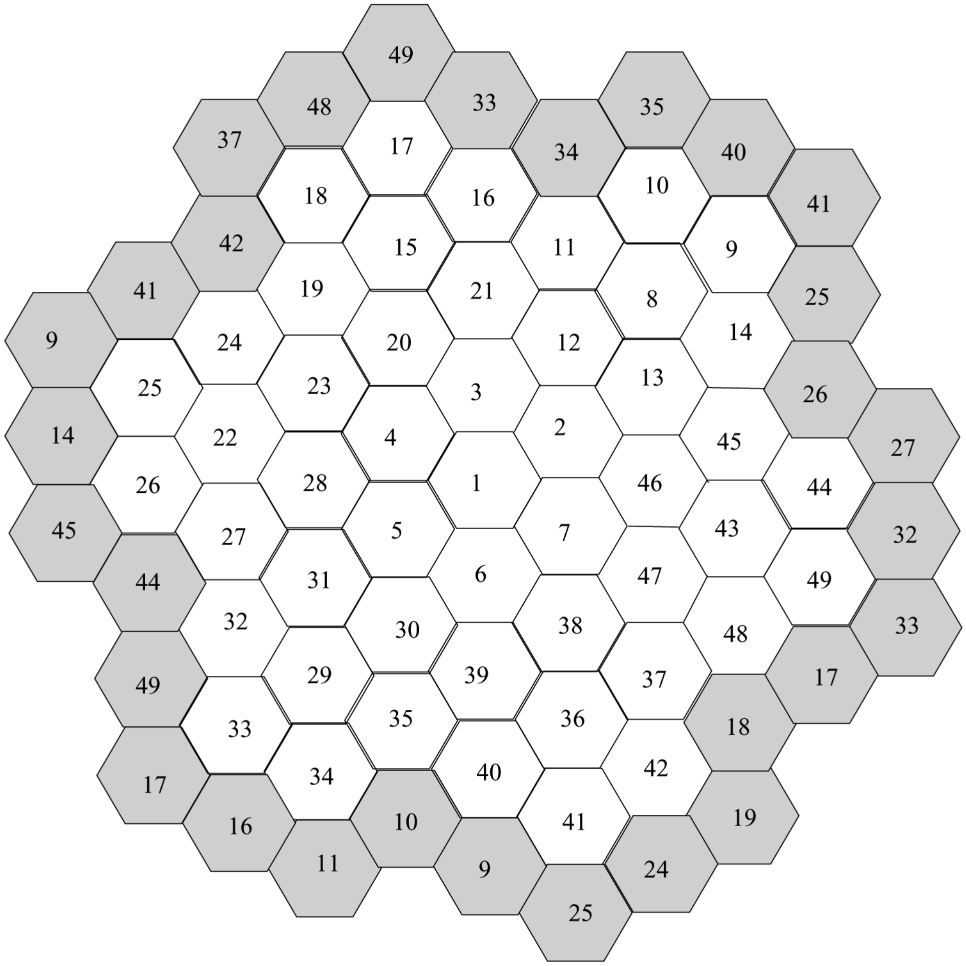

Fig. 2. The cell layout for the simulation model.

using a simple Brownian-motion or random-walk approximation [7,8]—a MT moves to any of the current cell's neighbors with equal probability of 1/6. Residence time of a MT in a cell, known as *dwell time,* is exponentially distributed with mean $1/\eta = 12$ s. New call arrivals follow a Poisson distribution with λ calls/s, and the call holding time or the total duration of the call follows an exponential distribution with mean $1/\mu = 120$ s. The *load* of a cell is the ratio of call arrival rate to call completion rate, $\rho = \lambda/\mu$ Erlangs/cell.

We have run simulation and the iterative algorithm with various combinations of different values of parameters. For each set of values of the parameters, the difference between data obtained from simulation and the iterative algorithm was insignificant. For these comparison tests, the precision of the numeric results was 0.001. Figure 3, a typical example, shows the plots of simulation and numerical data. It is seen that there is no significant difference between the simulation and iterative algorithm data. It is important to mention that the algorithm always converged.

5. OPTIMAL NUMBER OF GUARD CHANNELS

Now we use the preceding algorithm to find the optimal number of guard channels at which the supported new-call arrival rate reaches the maximum value, for given values of p_b and p_{ft}. Suppose the QoS requires that $p_b < p_{b0}$, $p_{ft} < p_{ft0}$, and let the desired precision be *accuracy*. The algorithm *Find_Best* computes the optimal load starting at zero load. Initially, the load value is incremented by a predefined constant (the constant value is 1 in our implementation) until we get two consecutive values of load, ρ_0*_lower_end* and ρ_0*_upper_end*, such that the load value ρ_0*_lower_end* satisfies both QoS conditions, but the load value ρ_0*_upper_end* does not satisfy one or both of the QoS conditions. The optimal value of load lies between these two extreme values. Now a binary search (procedure *Binary_Search_*ρ_0) on the segment [ρ_0*_lower_end*, ρ_0*_upper_end*] is performed until we get an optimal value of the load with the given accuracy.

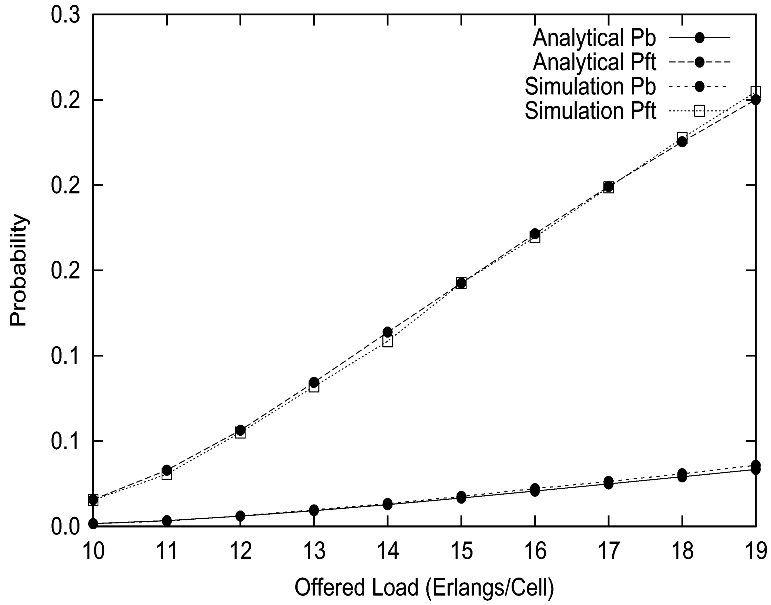

Fig. 3. A comparison of data from the iterative algorithm and simulation.

Procedure *Find_Best* (Input: s, μ, η, p_{ft0}, p_{b0}, *accuracy;* Output: *maximum_*ρ_0, *optimal_g*)
begin
  set $g = 0$, *optimal_g* $= 0$, $p_b = 0$;
  set *maximum_*$\rho_0 = -1$; // such that *maximum_*$\rho_0 < \rho_0$
  while ($g < s$ or $p_b < p_{b0}$) **do**
  begin
    begin // *Iterate_*ρ_0
      set $\rho_0 = 0$, $d\rho_0 = 1$, $p_{ft} = 0$, $p_{hf} = 0$, $p_b = 0$;
      while ($p_b \leq p_{b0}$ and $p_{ft} \leq p_{ft0}$) **do**
      begin
        set ρ_0*_lower_end* $= \rho_0$;
        increment ρ_0 by $d\rho_0$;
        Find_Probabilities (Input: ρ_0,*g;* Output: p_b, p_{ft});
        set ρ_0*_upper_end* $= \rho_0$;
      end;
    end;
    *Binary_Search_*ρ_0 (Input: ρ_0*_lower_end*, ρ_0*_upper_end;* Output: ρ_0);
    set ρ_0*_upper_end* $= \rho_0$;
    if (*maximum_*$\rho_0 < \rho_0$) **then**
    begin
      set *maximum_*$\rho_0 = \rho_0$, *optimal_g* $=$ *g;*
    end;
    increment g by 1;
  end;
  output (*maximum_*ρ_0, *optimal_g*);
end.

Here is the description of the recursive procedure *Binary_Search_*ρ_0.

Procedure *Binary_Search_* ρ_0(Input: ρ_0*_lower_end*, ρ_0*_upper_end;* Output: ρ_0)
begin
  if ((ρ_0*_upper_end* – ρ_0*_lower_end*) $>$ *accuracy*) **then**
  begin
    set $p_{ft} = 0$, $p_{hf} = 0$, $p_b = 0$;
    set ρ_0*_middle* $=$ (ρ_0*_upper_end* $+$ ρ_0*_lower_end*)/2;
    Find_Probabilities(Input: ρ_0*_middle, g;* Output: p_b, p_{ft});
    if ($p_b \leq p_{b0}$ and $p_{ft} \leq p_{ft0}$) **then**
      set ρ_0*_lower_end* $= \rho_0$*_middle;*
    else
      set ρ_0*_upper_end* $= \rho_0$*_middle;*
    *Binary_Search_*ρ_0(Input: ρ_0*_lower_end*, ρ_0*_upper_end;* Output: ρ_0);
  end;
  set $\rho_0 =$ (ρ_0*_upper_end* $+$ ρ_0*_lower_end*)/2;
end.

6. RESULTS AND DISCUSSION

The data obtained from the iterative algorithm presented in Section 5 were compared with those obtained from simulations. We present only a few representative

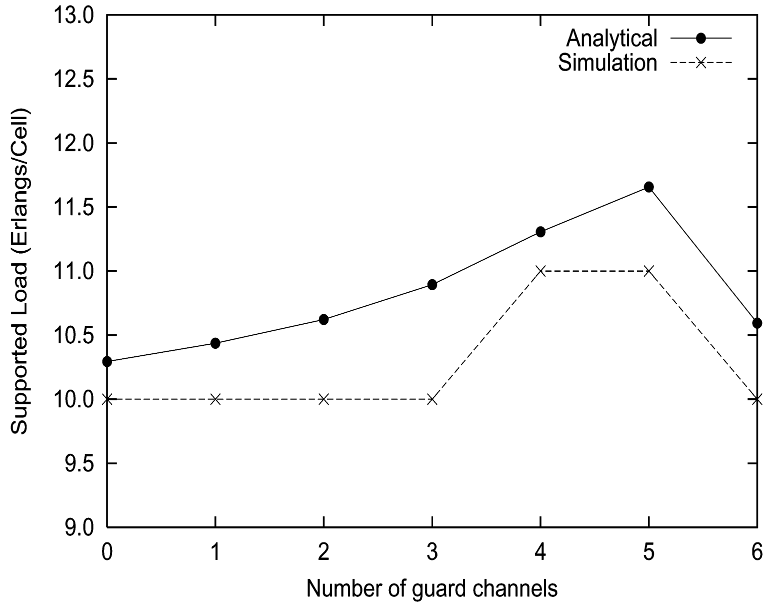

Fig. 4. A comparison of numerical and simulation results for $p_b = 0.1$, $p_{ft} = 0.02$.

samples. It may be recalled that the values of the parameters for the results reported are: number of channels in the cell is $s = 20$, an average holding time of calls is $1/\mu = 120$ seconds, an average dwell time of calls is $1/\eta = 12$ seconds, and the precision of numerical results is 0.001. Figures 4 and 5 are produced using data from the iterative algorithm and simulations; they suggest that for a given pair of values for p_b and p_{ft}, there is an optimal number of guard channels that supports maximum load (or new-call arrival rate). *The deviation of the plots is* not *a disagreement of simulation and numerical data; it is due to the increment of load by 1 at each simulation run.* We ran simulations for loads 9, 10, and 11 etc. until we found the load that would exceed one or both of the

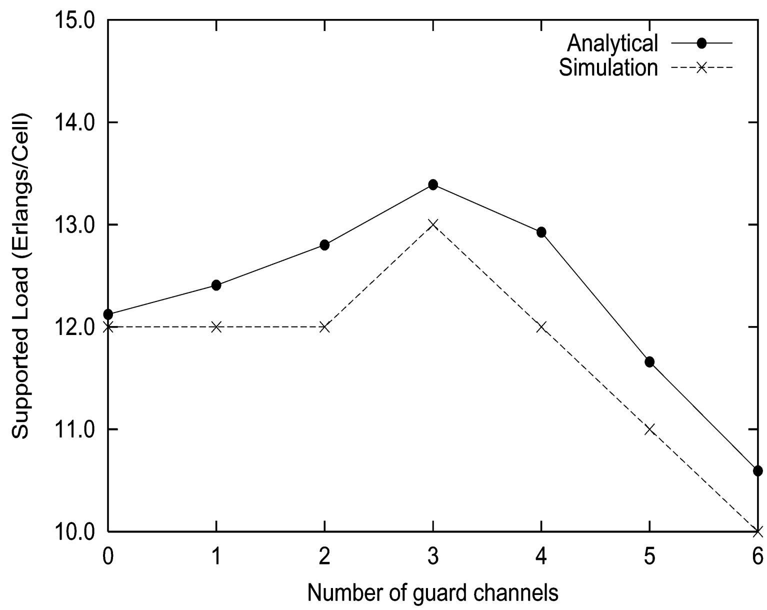

Fig. 5. A comparison of numerical and simulation results for $p_b = 0.1$, $p_{ft} = 0.06$.

QoS parameters. For instance, when the number of guard channels was three, a load of 11 exceeded the desired value of P_{ft}. Thus, the acceptable load from simulation was 10. If one is interested in finding the *maximum acceptable* load, the simulations could be run for several loads between 10 and 11. Actually, the value of the acceptable maximum load we obtained from our analytical model is what one would get. However, the computation time that would be necessary for these simulation runs is very long. In fact, our analytical model found all seven values of the acceptable maximum load in less time than that necessary for one single simulation run. Thus, the analytical model, when available, requires less computation time and involves very little programming time.

The algorithm presented in Section 5 finds the optimal number of guard channels by using the iterative algorithm described in Section 3. The value of the optimal number of guard channels is useful for efficient utilization of limited resources in a wireless network.

REFERENCES

1. A. S. Acampora and M. Naghshineh, Control and quality of service provisioning in high speed microcellular networks, *IEEE Personal Communications,* Vol. 1, No. 2, pp. 36–43, 1994.
2. L. O. Guerrero and A. H. Aghvami, A prioritized handoff dynamic channel allocation strategy for PCS, *IEEE Transactions on Vehicular Technology,* Vol. 48, No. 4, pp. 1203–1215, 1999.
3. D. Hong and S. S. Rappaport, Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures, *IEEE Transactions on Vehicular Technology,* Vol. 35, No. 3, pp. 77–92, 1986.
4. Y. C. Kim, D. E. Lee, B. J. Lee, Y. S. Kim, and B. Mukherjee, Dynamic channel reservation based on mobility in wireless atm network, *IEEE Communications Magazine,* Vol. 37, No. 11, pp. 47–51, 1999.
5. Y. B. Lin, S. Mohan, and A. Noerpel, PCS channel assignment strategies for hand-off and initial access, *IEEE Personal Communications,* Vol. 1, No. 3, pp. 47–56, 1994.
6. Y. B. Lin, S. Mohan, and A. Noerpel, Queuing priority channel assignment strategies for PCS hand-off and initial access, *IEEE Transactions on Vehicular Technology,* Vol. 43, No. 3, pp. 704–712, 1994.
7. Y. B. Lin, L. F. Chang, and A. Noerpel, Modeling hierarchical microcell/macrocell PCS architecture. In *Proceedings of ICC,* pp. 405–409, 1995.
8. A. D. May, *Traffic Flow Fundamentals,* Prentice Hall, Upper Saddle River, NJ 07458, 1990.
9. S. -H. Oh and D. -W. Tcha, Prioritized channel assignment in a cellular radio network, *IEEE Transactions on Communications,* Vol. 40, No. 7, pp. 1259–1269, 1992.
10. D. Towsley, R. Ramjee, and R. Nagarajan, On optimal call admission control in cellular networks, *Wireless Networks,* Vol. 3, pp. 29–41, 1997.
11. S. Tekinay and B. Jabbari, A measurement-based prioritization scheme for handovers in mobile cellular networks, *IEEE Journal on Selected Areas in Communications,* Vol. 10, No. 8, pp. 1343–1350, 1992.
12. O. T. W. Yu and V. C. M. Leung, Adaptive resource allocation for prioritized call admission over an ATM-based wireless-PCN, *IEEE Journal on Selected Areas in Communications,* Vol. 15, No. 7, pp. 1208–1225, 1997.

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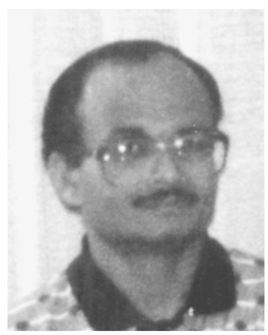

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