

# Logarithmic expected packet delivery delay in mobile ad hoc wireless networks

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## Summary

It has been shown that in a mobile ad hoc wireless network per node throughput remains constant as the number of nodes approaches infinity, when the nodes lie in a disk of unit area. However, this requires that some packets are delivered indirectly by a relay node. In this paper, we prove, in the same setup, that  $D(k) \in \Theta(\log k)$ , where  $D(k)$  is the expected delay time of a mobile ad hoc wireless network with  $k$  nodes. As consequence, we have concluded that mobile ad hoc wireless networks are not scalable for real-time applications. Copyright © 2004 John Wiley & Sons, Ltd.

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**KEY WORDS:** ad hoc wireless networks; Brownian motion; packet delivery; throughput; delivery delay

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## 1. Introduction

In Reference [6], M. Grossglauser and D. Tse studied the total throughput capacity for a model of an ad hoc wireless network, where  $k$  nodes, lying in a disk of unit area, communicate in random source–destination pairs. They have shown that when nodes are allowed to move, but communication between source and destination pairs is direct, per user throughput decreases as the number of nodes increases. However, if packets are relayed just once, then per node throughput becomes asymptotically constant as the number of nodes increases. They were aware of the fact that the cost to pay for the increased throughput was delay. They did not attempt to estimate delay. In this paper, we provide a tight bound estimate for the delay; we prove the following theorem.

**Theorem 1:** Let  $D(k)$  be the expected delay time, per packet, of a mobile ad hoc wireless network with  $k$  nodes lying on a Torus of unit area. Then  $D(k) \in \Theta(\log k)$ . Moreover

$$\lim_{k \rightarrow \infty} \frac{D(k)}{\log k} = \frac{1}{4\pi}$$

## 2. Preliminaries

### 2.1. Mobile Model

In this paper, we follow notation similar to that in Reference [6]. An ad hoc network consists of  $k$  nodes, all lying in a unitary square.<sup>‡</sup> Let the location of the  $i$ th node at time  $t$  be given by  $X_i(t)$ . We identify a node with its corresponding position on the unitary square.

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<sup>‡</sup> Observe that the rôle played by the unitary disk in Reference [6] is not of importance for their results. For example, the estimate  $\lim_{z \rightarrow 0} F(z)/z^{2/\alpha} = \pi$ , obtained in Reference [6], does not depend on the fact that the nodes lie on the unitary disk. This estimate only depends on the fact that the measure used on the unitary disk is the restriction of the Lebesgue measure. The same estimate can be obtained for the unitary square, see Reference [11].

Nodes are mobile and we assume that the process induced by every node,  $\{X_i(\cdot)\}$ , is stationary and ergodic with stationary distribution uniform on the unitary square; moreover, the trajectories of different nodes are independent and identically distributed.

We assume that each of the  $k$  nodes is a source node for one session and a destination node for another session. We assume that each source node has an infinite stream of packets to send to its destination. The pairing will not change with time.

For a given  $t$ , we denote by  $P_i(t)$  the transmission power of node  $i$ , and by  $\gamma_{ij}(t)$  the channel gain from node  $i$  to node  $j$ . Moreover, node  $i$  will transmit data at rate  $R$  packets/s to node  $j$  if

$$\frac{P_i(t)\gamma_{ij}(t)}{1/L \sum_{k \neq i} P_k(t)\gamma_{ki}(t) + N_0} > \beta \quad (1)$$

where  $\beta$  is the signal-to-interference ratio (SIR) requirement for a successful communication.  $N_0$  is the background noise power and  $L$  is the processing gain of the system. In this paper, we assume the same channel model as in Reference [6], i.e., the channel gain from node  $i$  to node  $j$  is given by

$$\gamma_{ij}(t) = d(X_i(t), X_j(t))^{-\alpha}$$

where  $d(X_i(t), X_j(t))$  denotes the distance between node  $i$  and node  $j$  at time  $t$ , and  $\alpha \geq 2$ . Packet transmission can be direct or through one or more nodes serving as relays. We assume each node has an infinite buffer to store relayed packets.

### 2.2. Mobile Nodes with Relaying and Scheduling Policy

In Reference [6], it was shown that an asymptotic constant throughput can be obtained by relaying the packets only once. To deal with relaying, a scheduling policy is mandatory.

For a given  $\theta \in (0, 1)$ , the sender density parameter, they proposed the following scheduling policy  $\prod_\theta$ : for every  $t$ ,  $k_S = \lfloor k\theta \rfloor$  nodes are randomly chosen as the senders, and the remaining  $k_R$  nodes as the potential receivers. Each sender node transmits packets to its nearest neighbor, among all potential receivers, using unit transmit power. Among the sender–receiver pairs, we only choose those pairs for which the interference generated by the other senders is sufficiently small so that transmission is possible. Let  $N_t(k)$  be the number of such pairs, under this scheduling policy the number of feasible sender–receiver pairs  $N_t(k) \in \Theta(k)$ .

**Theorem** (M. Grossglauser, D. Tse): *For the scheduling policy  $\prod_\theta$ , the expected number of feasible sender–receiver pairs  $\mathbb{E}[N_t(k)]$ , satisfies the following asymptotic:*

$$\lim_{k \rightarrow \infty} \frac{\mathbb{E}[N_t(k)]}{k} = \phi(\theta) > 0.$$

Furthermore, for any two arbitrary nodes  $i$  and  $j$ , the probability that the pair  $(i, j)$  is scheduled as a sender–receiver pair is  $\Theta(1/k)$ .

If we want to estimate the expected delay time, per packet, as a function of the number of nodes, we have to introduce a mobility model. Such a mobility model has to take into consideration the boundary of our domain. In order to specify a mobility model, it is assumed that the nodes follow a Brownian motion on a at 2-torus  $(T^2, g_0)$ .

### 2.3. Flat 2-Torus $(T^2, g_0)$

We have decided to use as our domain model the flat 2-torus, because it is easier to estimate the distance between two points in this model than in any other model, and this is important for the simulation. Our results do not depend on this fact (see footnote on page 14).

Let  $\mathbb{R}^2$  be a plane with coordinates  $(x, y)$  and  $T_{m,n} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the translation  $T_{m,n}(x, y) = (x + m, y + n)$ , where  $m, n \in \mathbb{Z}$ . Define an equivalence relation in  $\mathbb{R}^2$  by  $(x, y) \sim (x_1, y_1)$ , if there exist integers  $m, n$  such that  $T_{m,n}(x, y) = (x_1, y_1)$ . Let  $T^2$  be the quotient space of  $\mathbb{R}^2$  by this equivalence relation, that is  $\mathbb{R}^2 / \mathbb{Z}^2$ . Thus, in each open unit square whose vertices have integer coordinates, there is only one representative of  $T^2$ , and  $T^2$  may be thought of as a closed square with opposite sides identified. See Figure 1.

Since translations on  $\mathbb{R}^2$  are isometries, the Euclidean metric on  $\mathbb{R}^2$  descends to the quotient  $T^2$ .

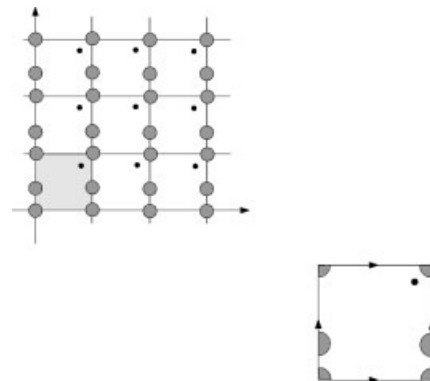


Fig. 1. Flat torus.

In Figure 1, we have drawn  $\epsilon$ -neighborhoods of points in the boundary. An  $\epsilon$ -neighborhood of a point far from the boundary looks the same way as in  $\mathbb{R}^2$ .

The distance,  $d_{T^2}([x_1, y_1], [x_2, y_2])$ , between two points  $[x_1, y_1]$  and  $[x_2, y_2]$  in  $T^2$  is given by:

$$\inf_{m,n \in \mathbb{Z}} \left\{ \sqrt{|x_1 - x_2 + m|^2 + |y_1 - y_2 + n|^2} \right\}$$

### 3. Brownian Motion on Riemannian Manifolds

We do not pretend to have a crash course in Riemannian geometry nor in Brownian motion in this paper, the reader is directed to the following references, and references therein, where the concepts are explained in detail, [5],[1], [8], [7] and [2].

**Definition 1:** Let  $(M^n, g_0)$  be a closed Riemannian manifold of dimension  $n$ , and  $\epsilon > 0$ . We say that  $x, y \in M$  are  $\epsilon$ -close if  $d(x, y) < \epsilon$ . Here  $d : M \times M \rightarrow \mathbb{R}^+$ , denotes the distance function induced by the Riemannian metric  $g_0$  on  $M$ .

**Definition 2:** Let  $(M^n, g_0)$  be a closed Riemannian manifold of dimension  $n \geq 2$ , and  $\epsilon > 0$ . Given two independent Brownian motions  $X_t, Y_t$  on  $(M^n, g_0)$ , the  $\epsilon$ -meeting time  $T_\epsilon$  is defined by:<sup>§</sup>

$$T_\epsilon(\omega) = \inf\{t \mid d(X_t(\omega), Y_t(\omega)) < \epsilon\}$$

**Example 1:** Since it is always useful to have a picture in mind, let us assume that  $X_t$  and  $Y_t$  are two independent Brownian motions on the unit circle  $\mathbb{S}^1$ , thought of as  $\mathbb{R}/\mathbb{Z}$ , and we want to know, what is the  $\epsilon$ -meeting time for this two Brownian motions?

In Figure 2,  $X_s, Y_s$  and  $X_t, Y_t$  represent the position of two Brownian motions at time  $s$  and  $t$  respectively. We have drawn them in different lines (open circles) to represent that they are independent. Remember that the endpoints are identified.

The idea of the proof of Theorem 3 is the following: instead of having two Brownian motions  $X_t$  and  $Y_t$  on  $\mathbb{S}^1$ , work with the Brownian motion<sup>¶</sup>  $(X_t, Y_t)$  on  $\mathbb{S}^1 \times \mathbb{S}^1$ . The question when  $X_t$  and  $Y_t$  are  $\epsilon$ -close on  $\mathbb{S}^1$  has to have a counterpart on  $\mathbb{S}^1 \times \mathbb{S}^1$ .

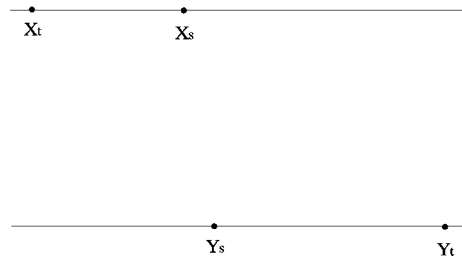


Fig. 2. Independent Brownian motions on  $\mathbb{S}^1$ .

In this context, the interpretation of Lemma 1 is the following: two points  $x, y \in \mathbb{S}^1$  are  $\sqrt{2}\epsilon'$ -close if and only if the pair  $(x, y) \in \mathbb{S}^1 \times \mathbb{S}^1$  lies within a band of width  $2\epsilon'$  centered at the diagonal,<sup>||</sup> see Figure 3.

**Lemma 1:** Let  $(M^n, g_0)$  be a Riemannian manifold,  $(N, g) = (M, g_0) \times (M, g_0)$ ,  $\Delta_M$  be the image of the diagonal map

$$\begin{aligned} \Delta : M &\rightarrow M \times M = N \\ x &\rightarrow (x, x) \end{aligned}$$

and  $\mathcal{N}_{\epsilon'}\Delta_M \subset N$  be an  $\epsilon'$ -tubular neighborhood of  $\Delta_M$ . Then for sufficiently small  $\epsilon'$

$$\mathcal{N}_{\epsilon'}\Delta_M = d^{-1}([0, \sqrt{2}\epsilon'])$$

that is, two points  $x, y \in M$  are  $\sqrt{2}\epsilon'$ -close if and only if  $(x, y) \in \mathcal{N}_{\epsilon'}\Delta_M$ .

*Proof.* Let us show that  $\mathcal{N}_{\epsilon'}\Delta_M \subset d^{-1}([0, \sqrt{2}\epsilon'])$ , where  $\sqrt{2}\epsilon' < \min(\text{inj}(M), \text{inj}(N))$ .<sup>\*\*</sup> Let  $(x, y) \in \mathcal{N}_{\epsilon'}\Delta_M \setminus \Delta_M$ ,<sup>††</sup>  $(u, u) \in \Delta_M$  be the closest point on

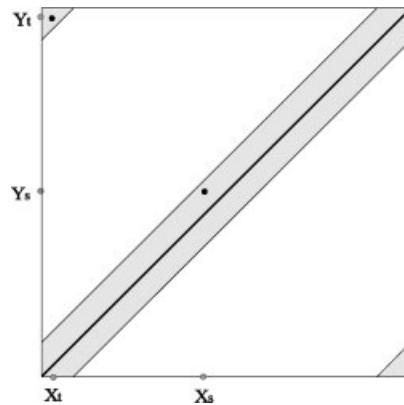


Fig. 3. Induced Brownian motion on  $T^2$ .

<sup>§</sup>The  $\epsilon$ -meeting time is the minimum time required for two independent Brownian motions to be  $\epsilon$ -close.

<sup>¶</sup>Any pair of independent Brownian motions on a Riemannian manifold, induce a Brownian motion on the Riemannian product manifold.

<sup>||</sup>In Riemannian geometry, this is called an  $\epsilon'$ -tubular neighborhood of the diagonal.

<sup>\*\*</sup> $\text{inj}(\cdot)$  denotes the injectivity radius, see References [1] and [5].

<sup>††</sup> $A \setminus B = A \cap B^c$ .

$\Delta_M$  to  $(x, y)$  and  $\bar{\gamma}(t) = (\alpha(t), \beta(t))$  be the unique geodesic on  $N$ , starting at  $(x, y)$  and ending at  $(u, u)$ , where  $t \in [0, \ell]$ . Since  $\bar{\gamma}$  realizes the path of minimal distance between  $(x, y)$  and  $\Delta_M$  then  $\bar{\gamma}'(\ell^-) \in T_{(u,u)}\Delta_M$ , in other words

$$\lim_{t \rightarrow \ell^-} \alpha'(t) = \lim_{t \rightarrow \ell^-} \beta'(t) \tag{2}$$

Since  $\bar{\gamma}$  is a geodesic on  $N$ , then for every  $t$ ,  $|\alpha'(t)|_M^2 = c$ , and  $|\beta'(t)|_M^2 = 1 - c$ . Using Equation (2), we obtain that  $c = 1/2$ . It is easy to see that  $\gamma(t)$  defined by

$$\gamma(t) = \begin{cases} \alpha(\sqrt{2}t) & \text{if } t \in \left[0, \frac{\ell}{\sqrt{2}}\right) \\ \beta(2\ell - \sqrt{2}t) & \text{if } t \in \left[\frac{\ell}{\sqrt{2}}, \sqrt{2}\ell\right] \end{cases}$$

is a geodesic on  $M$ , starting at  $x$  and ending at  $y$ . Hence  $d_M(x, y) \leq \sqrt{2}\ell < \sqrt{2}\epsilon'$ .

To show that  $d^{-1}([0, \sqrt{2}\epsilon']) \subset \mathcal{N}_{\epsilon'}\Delta_M$ , where  $\sqrt{2}\epsilon' < \min(\text{inj}(M), \text{inj}(N))$ , let  $x, y \in M$ , such that  $d(x, y) = L < \sqrt{2}\epsilon'$ , and  $\gamma$  be the unique geodesic on  $M$  starting at  $x$  and ending at  $y$ , where  $t \in [0, L]$ . Since  $|\gamma'(t)| = 1$  for every  $t$ , then  $\bar{\gamma}(t) = (\gamma(t/\sqrt{2}), \gamma(L - t/\sqrt{2}))$  is a geodesic on  $N$ , starting at  $(x, y)$  and ending at  $\Delta_M$ , where  $t \in [0, L/\sqrt{2}]$ . Hence  $(x, y) \in \mathcal{N}_{L/\sqrt{2}}\Delta_M \subset \mathcal{N}_{\sqrt{2}\epsilon'/\sqrt{2}}\Delta_M = \mathcal{N}_{\epsilon'}\Delta_M$ .  $\square$

Lemma 1 allow us to restate our question: What is the expected time for a Brownian motion on a closed Riemannian manifold, to lie within an  $\epsilon$ -tubular neighborhood of a submanifold? This question has been studied by mathematicians—see the proof of proposition 2 and references therein. Theorem 3 summarizes all of the above and gives us the answer we were looking for.

**Proposition 2:** *Let  $(N^m, g)$  be a closed Riemannian manifold of dimension  $m \geq 2$  and  $\Delta \subset N$  a submanifold of codimension  $q \geq 2$ . If  $\tau_{\epsilon}$  denotes the first exit time of a Brownian motion on  $(N^m, g)$  from  $N \setminus \mathcal{N}_{\epsilon}\Delta$ , then*

$$\lim_{\epsilon' \rightarrow 0} \phi_p(\epsilon') \int_N \mathbb{E}^z[\tau_{\epsilon'}] d\mu_g = 2 \frac{\text{Vol}(N)^2}{\text{Vol}(\Delta)}$$

where

$$\phi_p(\epsilon') = \begin{cases} 2\pi |\log \epsilon'|^{-1} & \text{if } p = 2 \\ (p - 2)\text{Vol}(\mathbb{S}^{p-1})\epsilon'^{p-2} & \text{if } p \geq 3 \end{cases}$$

*Proof.* First, we will prove the following inequality

$$\lim_{\epsilon' \rightarrow 0} \phi_p(\epsilon') \int_N \mathbb{E}^z[\tau_{\epsilon'}] d\mu_g \leq 2 \frac{\text{Vol}(N)^2}{\text{Vol}(\Delta)}$$

Let us make  $D = N \setminus \mathcal{N}_{\epsilon'}\Delta$ . A consequence of the work of P. McDonald and R. Meyers is the following upper bound for  $\int_N \mathbb{E}^x[\tau_{\epsilon'}] d\mu_g$ , see Reference [10]

$$\int_N \mathbb{E}^z[\tau_{\epsilon'}] d\mu_g \leq 2 \frac{\text{Vol}(N \setminus \mathcal{N}_{\epsilon'}\Delta)}{\lambda_1(N \setminus \mathcal{N}_{\epsilon'}\Delta)}$$

After multiplying both sides by  $\phi_p(\epsilon')$  and using that

$$\lim_{\epsilon' \rightarrow 0} \frac{\lambda_1(N \setminus \mathcal{N}_{\epsilon'}\Delta)}{\phi_p(\epsilon')} = \frac{\text{Vol}(\Delta)}{\text{Vol}(N)}$$

where

$$\phi_p(\epsilon') = \begin{cases} 2\pi |\log \epsilon'|^{-1} & \text{if } p = 2 \\ (p - 2)\text{Vol}(\mathbb{S}^{p-1})\epsilon'^{p-2} & \text{if } p \geq 3 \end{cases}$$

which is proved by G. Curtois in Reference [3], we obtain

$$\lim_{\epsilon' \rightarrow 0} \phi_p(\epsilon') \int_N \mathbb{E}^z[\tau_{\epsilon'}] d\mu_g \leq 2 \frac{\text{Vol}(N)^2}{\text{Vol}(\Delta)}$$

The other inequality

$$\lim_{\epsilon \rightarrow 0} \phi_p(\epsilon') \int_N \mathbb{E}^z[\tau_{\epsilon'}] d\mu_g \geq 2 \frac{\text{Vol}(N)^2}{\text{Vol}(\Delta)}$$

was just recently proved by one of the authors, see Reference [4].  $\square$

**Theorem 3:** *Let  $(M^n, g_0)$  be a closed Riemannian manifold of dimension  $n \geq 2$ ,  $X_t$  and  $Y_t$  be independent Brownian motions on  $(M, g_0)$ ,  $\epsilon > 0$ , and  $T(\epsilon)$  be the expected  $\epsilon$ -meeting time for  $X_t$  and  $Y_t$ . Then*

$$\lim_{\epsilon \rightarrow 0} \frac{T(\epsilon)}{\psi_n(\epsilon)} = d_n \text{Vol}(M)$$

where

$$\psi_n(\epsilon) = \begin{cases} |\log \epsilon| & \text{if } n = 2 \\ \epsilon^{2-n} & \text{if } n \geq 3 \end{cases}$$

and

$$d_n = \begin{cases} \frac{1}{2\pi} & \text{if } n = 2 \\ \frac{2\Gamma(n/2)}{(n-2)(4\pi)^{n/2}} & \text{if } n \geq 3 \end{cases}$$

*Proof.* Let  $x, y \in M$ , and consider two independent Brownian motions  $X_t, Y_t$  on  $(M^n, g_0)$ , such that  $\mathbb{P}(X_0 = x) = \mathbb{P}(Y_0 = y) = 1$ . If

$$t_{x,y}(\epsilon) = \inf\{t > 0 \mid d(X_t, Y_t) < \epsilon\}$$

then

$$T(\epsilon) = \frac{1}{\text{Vol}^2(M)} \iint_{M \times M} \mathbb{E}[t_{x,y}(\epsilon)] d\mu_{g_0} d\mu_{g_0}$$

Making  $\epsilon' = \epsilon/\sqrt{2}$  in Lemma 1, we will have that two points  $x, y \in M$  are  $\epsilon$ -close if and only if  $(x, y) \in \mathcal{N}_{\epsilon/\sqrt{2}}\Delta_M$ , therefore

$$t_{x,y}(\epsilon) = \inf\{t > 0 \mid Z_t \in \mathcal{N}_{\epsilon/\sqrt{2}}\Delta_M\}$$

where  $Z_t = (X_t, Y_t)$ ,  $z = (x, y)$ , and

$$\mathbb{P}(Z_0 = z) = 1$$

Since  $X_t$  and  $Y_t$  are independent Brownian motions on  $(M, g_0)$  then  $Z_t = (X_t, Y_t)$  is a Brownian motion on  $(N, g) = (M \times M, g_0 + g_0)$  and the above identity for  $t_{x,y}(\epsilon)$  can be expressed in terms of  $\tau_\epsilon$ , the first exit time of a Brownian motion  $Z_t$ , on  $N$  from  $N \setminus \mathcal{N}_\epsilon \Delta_M$ , where  $\mathbb{P}\{Z_0 = z\} = 1$

$$\mathbb{E}[t_{x,y}(\epsilon)] = \mathbb{E}^z[\tau_{\epsilon/\sqrt{2}}]$$

As a consequence of this, we have that

$$T(\epsilon) = \frac{1}{\text{Vol}^2(M)} \int_{M \times M} \mathbb{E}^z[\tau_{\epsilon/\sqrt{2}}] d\mu_g$$

Since  $\Delta_M \subset N$  is a submanifold of codimension  $n$  and  $\epsilon' = \epsilon/\sqrt{2}$  we have that  $\phi_n(\epsilon/\sqrt{2}) = 2^{(2-n/2)}/d_n \phi_n(\epsilon)$ . Here we have used the identity  $\text{Vol}(\mathbb{S}^{n-1}) = 2\pi^{n/2}/\Gamma(n/2)$ . Furthermore, the identities  $\text{Vol}(N) = \text{Vol}(M)^2$  and  $\text{Vol}(\Delta_M) = 2^{n/2} \text{Vol}(M)$  can be easily obtained using that  $(N, g) = (M \times M, g_0 + g_0)$  is the Riemannian product manifold. Putting all these together and the above result we obtain

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{T(\epsilon)}{\psi_n(\epsilon)} &= 2^{(n-2)/2} d_n \frac{2 \text{Vol}(M)}{2^{n/2}} \\ &= d_n \text{Vol}(M) \end{aligned}$$

## 4. Main Result

In order to achieve the desired throughput, M. Grossglauser and D. Tse have shown in Reference [6], the necessity of relaying the data packets; in fact, the desired throughput can be achieved using only one relay. They also have shown that any pair of nodes  $(i, j)$  is equally likely, in the limit, to be chosen as a feasible sender–receiver pair. Therefore, in order to compute the delay time, per packet, we need only to do the computation for a randomly chosen pair  $(i, j)$ .

How close should sender and receiver be? Assume, for simplicity, that every sender node transmits at the same power  $P$ . Let us focus on the transmission from node  $i$  to  $j$ . From Equation (1), it can be seen that transmission from  $i$  to  $j$  will be unsuccessful whenever there is another transmitting interferer  $\ell$ , satisfying

$$|X_\ell - X_j| \leq \left(\frac{\beta}{L}\right)^{1/\alpha} |X_i - X_j|$$

Therefore, the best we can do is to restrict transmissions to neighbors, which are at a typical distance of  $1/\sqrt{k}$ .

**Theorem A:** *Let  $D(k)$  be the delay time, per packet, of a mobile ad hoc wireless network with  $k$  nodes lying on a Torus of unit area. Then  $D(k) \in \Theta(\log k)$ . Moreover*

$$\lim_{k \rightarrow \infty} \frac{D(k)}{\log k} = \frac{1}{4\pi}$$

*Proof.* By the discussion above we are interested on finding how  $T(1/\sqrt{k})$  behaves for  $k$  big enough, when the nodes lie on a 2-torus<sup>††</sup> of unit area. In this case, we have that  $M^n = T^2$  and  $g_0$  is the at metric of unit area on  $T^2$ . Again by the discussion above, the delay time  $D(k)$ , per packet, of a mobile ad hoc wireless network with  $k$  nodes is given by the expected  $\epsilon$ -meeting time  $T(\epsilon)$ , of two Brownian motions on the flat 2-torus, with  $\epsilon = 1/\sqrt{k}$ . In particular

$$\lim_{k \rightarrow \infty} \frac{D(k)}{\log k} = \lim_{k \rightarrow \infty} \frac{T(1/\sqrt{k})}{2 \log \sqrt{k}}$$

<sup>††</sup>Theorem 3 shows that this estimate does not depend on the way we choose to get rid of the boundary as long as we end up considering a smooth closed surface without boundary as our domain.

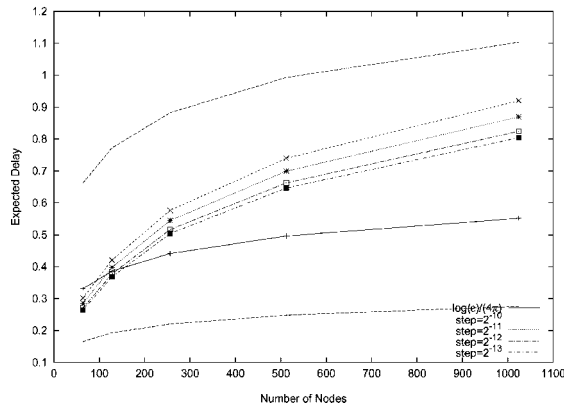


Fig. 4. Simulation results.

Now using Theorem 3, with  $n=2$  and  $\epsilon = 1/\sqrt{k}$  we obtain

$$\lim_{k \rightarrow \infty} \frac{T(1/\sqrt{k})}{2 \log \sqrt{k}} = \frac{1}{2} d_2 \text{Vol}(T^2)$$

since  $\text{Vol}(T^2) = 1$  and  $d_2 = 1/2\pi$ , we obtain

$$\lim_{k \rightarrow \infty} \frac{D(k)}{\log k} = \frac{1}{4\pi}$$

## 5. Simulation Results

We have used Paxson method to approximate Brownian motions [12], and the Mersenne–Twister pseudo random number generator [9]. Our simulation results are computed using 1000 runs (to discretize integration) for 64, 128, 256, 512 and 1024 nodes.

Figure 4 shows how the simulation results approach the theoretical limit. We have plotted four sets of simulations. As we decrease the step size, the obtained delays approaches our limit. The outermost curves are simply the double and half of the theoretical upper bound.

## 6. Conclusions

Advancement in hardware and software technologies has enabled us to deploy ad hoc networks. Nodes in

an ad hoc network may be mobile and static. It had been shown that per node throughput of a mobile ad-hoc network goes to zero as the number of nodes goes to infinity. Per node throughput does not increase when nodes can move, but packet exchanges between source–destination pairs must occur directly. However, when a packet is exchanged, using at most one relay, per node throughput increases dramatically to a constant, as the number of nodes goes to infinity. It has been stated that this gain in throughput could increase delay [6]. However, to the best of our knowledge, no estimate for the expected delay per packet is known. In this paper, we provide a tight bound for the delay, we show that delay has logarithmic growth. As consequence we have that mobile ad-hoc wireless networks are not scalable for real-time applications.

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## Authors' Biographies



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