Modeling Full-Length Video Using Markov-Modulated Gamma-Based Framework

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Abstract— **All traffic models for MPEG-like encoded *variable bit rate* (VBR) video can be categorized into (*i*) *data rate models* (DRMs), and (*ii*) *frame size models* (FSMs). Almost all proposed VBR traffic models are DRMs. Since DRMs generate *only* data arrival rate, they are good for estimating average packet-loss and ATM buffer over-flowing probabilities, but fail to identify such details as percentage of frames affected. FSMs generate sizes of individual MPEG frames, and are good for studying frame loss rate in addition to data loss rate. Among three proposed FSMs: (*i*) one generates frame sizes for full-length movies without preserving GOP-periodicity; (*ii*) one generates VBR video traffic for news videos from scene content description provided to it; (*iii*) one generates frame sizes for full-length movies without preserving size-based video-segment transitions. In this paper we propose two FSMs that generate frame sizes for full-length VBR videos preserving both GOP-periodicity and size-based video-segment transitions.**

First, two-pass algorithms for analysis of full-length VBR videos are presented. After two-pass analysis these algorithms identify and partition (size-based) classes of video segments. Frames in each segment class produce three data-sets one each for I-, B-, and P-type frames. Each of these data-sets is modeled with an *axis shifted* Gamma distribution. Markov renewal processes model (size-based) video segment transitions. We have used QQ plots to show visual similarity of model-generated VBR video data-sets with original data-set. Leaky-bucket simulation study is used to show similarity of data and frame loss rates between model-generated VBR videos and original video. Our study of frame-based VBR video revealed even a low data loss rate could affect a large fraction of I frames causing a significant degradation of the quality of transmitted video.

I. Introduction

Video traffic is expected to be the major sources for *broadband integrated services digital networks* (B-ISDN) [MCS99], [CR99]. Because of large bandwidth requirement for communication of high-quality uncompressed video over B-ISDN, it is expected that most, if not all, video will be encoded with MPEG-like data compression techniques [FNQ00]. These compression algorithms can provide very high compression ratio while maintaining good quality of decompressed video. However, MPEG-coding provides different amount of compression for different frames (see Fig. 1 and Table 1 in [FNQ00], for examples), and results in *variable bit rate* (VBR) data — known as VBR video.

Accurate traffic models of VBR video is necessary for prediction of performance of any proposed (and/or designed) B-ISDN during its operation. Several traffic models have been proposed in the literature. They include first-order *autoregressive* (AR) [NFO89], *discrete* AR (DAR) [HTL92], [LNR94], [WCJ95], *Markov renewal process* (MRP) [DG99], MRP *transform-expand-sample* (TES) [MP98], *finite-state Markov chain* (MC)[HTL92], [WCJ95], [CR99], [RK98], *Gamma-beta auto-regression* (GBAR)[Hey97], and *group-of-pictures* (GOP) GBAR [FNQ00] models.

These traffic models can broadly be classified into two categories: (*i*) *data rate models* (DRMs), and (*ii*) *frame size models* (FSMs). In a data rate model, only the rate at which data are arriving at a link is generated for performance prediction purpose. Almost all models — including AR, DAR, MRP, MRP TES, MC, GBAR, and models in [HL96], [KM98] — fall under this category. These models are good for predicting average packet-loss probability, and ATM buffer over-flowing probability. However, they fail to identify such details as percentage of frames affected. Even a small rate of data loss may affect perceptual quality of received video significantly. Consider an MPEG encoded video with 15 frames per GOP, and 10^{-4} BLR affecting only 1% frames of all video frames. If all of those affected frames are I frames, 15% of the video will be affected— a perceptually significant degradation of video quality.

In a frame size model, sizes of individual MPEG frames are generated, and hence data rate information can be obtained from the frame size information. Models reported in [KT97], [DG99], and [FNQ00] fall under this category. Compared to number of DRMs, only few FSMs have been proposed. It is believed that the main obstacle in the development of an FSM is to find some standard statistical distribution fitting different frame types.

The GOP GBAR model attempts to capture overall statistical properties of I, P, and B frames of MPEG movies. However, this model does not attempt to capture shot-level regularity of sizes of I, P, and B frames, which is a typical characteristic of contents of any long video or any full-length movie [DG99]. The model in [KT97] assumes that the change of a scene changes the average size of I frames, but not the sizes of P and B frames. The Table IV in [DG99] shows average sizes of P and B frames can vary 20% and 30% respectively, which are statistically significant; our analysis of frames from a full-length video shows even wider variations (see Table I in Section II-C). Also, the sizes of the frames, drawn from log-normal distributions following [KSH95], are not quite good fits[1]. The model in [DG99] requires that the video to be modeled has been segmented into shots based on texture and motion. Moreover, it uses an AR model which has been reported to be unsatisfactory for full-length movies (see [FNQ00]).

In the next section we present our full-length video analysis. In Section III we present two models, one for each inter-class transition matrix. For validation of these models, full-length *synthetic* movies are generated using the proposed models. In Section IV, using two standard measures — QQ plot, and leaky bucket simulation — the models are validated. Only the results for the full-length movie *Crocodile*

[1] It must be acknowledged that one distribution is yet to be found that fits all frames of either I or B type of a movie well.

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Dundee[2] are reported in this paper. We discuss our observations, possible uses of proposed VBR video models, and future extensions of our work in Section V.

II. Analysis of VBR Video

This section outlines our analysis-algorithms of a full-length VBR video for extracting parameters to model it. Let $F = F_1, F_2, F_3, ...F_{n_f}$ be a sequence of n_f frames obtained from MPEG encoding of a full length video. Since, estimation of *data loss rate* (DLR) requires only the size of a frame and not the actual data, a frame for the modeling purpose is represented by its *serial number*, *type* (I, B, or P), and *size* or number of bits after MPEG encoding. While not required by MPEG standard, the underlying GOP usually follows a (N, M) cyclic format; the first frame of a GOP is a I frame, every M*-th* frame is a P frame, and the rest are B frames. We denote successive GOPs by $G_1, G_2, \ldots, G_{n_g}$ for our reference in the paper.

A. Formation of Clips

A *clip* of length k is any consecutive sequence of k GOPs, $k \geq 1$, that is, G_{i+1}, G_{i+2}, $\ldots, G_{i+k}$ for some $i, 0 \leq i \leq (n_f - k + 1)$. We denote successive clips by $C_1, C_2, \ldots$ and a set of clips by C. We use the notation $G_j \in C_i$ to indicate G_j belongs to C_i, and *length*(C_i) to indicate the number of GOPs in C_i.

We group 'similar-size' GOPs to obtain a set of video clips. During clip construction, let an average size of a GOP in the partially-formed clip of length k be, *clip_avg*= $(\sum_{l=1}^{k} size(G_{i+l}))/k$. The next GOP, G_{i+k+1}, is included in the current partial clip if its size does not differ from *clip_avg* by more than a user provided *threshold* value. The smaller the value of *threshold* the higher the number of clips formed. For the movies considered in this work, a value of 5000 was found quite satisfactory. However, the choice of threshold is not very critical for the modeling and any other value close to the average size of a *B*-frame could as well be used. The pseudo-code below explains the method.

```
Algorithm Clips_From_GOPs
Input – G : set of GOP;
  threshold : user provided value;
Output – C : set of clips;
begin
 insert G_1 into C_1;
 j = 1; /*index of the current clip*/
 clip_avg = size(G_1);
  /*average size of GOP in current clip*/
 for(i = 2; i ≤ n_g; i++){
  if abs(clip_avg − size(G_i)) ≤ threshold {
     /*continue expanding the current clip*/
   insert G_i into C_j;
   recompute clip_avg after inserting G_i in C_j;
  } else {  /*begin a new clip starting with G_i */
   j = j + 1;
   insert G_i into C_j;
   clip_avg = size(G_i);
  }
 } /*for */
end;
```

[2]The authors thank Wu-chi Feng for the MPEG traces.

The next step in our VBR video data analysis, outlined next, is partitioning of the clips into shot-classes.

B. Formation of Shot Classes from Clips

A *shot class* of length k, $k \geq 1$, is a union of k *distinct*, not necessarily consecutive, clips. We represent shot classes by $S_1, S_2, \ldots$ and $C_j \in S_i$ implies that all frames belonging to C_j belong to S_i. Every clip belongs to one and only one shot class.

We construct the shot classes by partitioning the range of GOP sizes into a desired number, say n, ($n = 7$ in our study) of shot classes. This is done by partitioning the entire range of GOP sizes, that is, the interval $[min_i\{size(G_i)\}, max_i\{size(G_i)\}]$ into n intervals, one for each shot class. *The boundaries of the intervals for successive shot classes were in an increasing geometric progression.* It may be noted a partitioning with subintervals of equal length produced rather unsatisfactory results.

The I, B, and P frames in the GOPs of a shot class j are separated to obtain S_{jI}, S_{jB}, and S_{jP}. Thus, the algorithm partitions the frames into $3n$ datasets. As discussed next, these datasets could be modeled quite accurately.

TABLE I

I Frame Characteristics in all Shot Classes

shot class	frame type	Parameters			
		offset(u)	μ	σ	#frame
1	I	10708	19657.95	2120.20	1870
2	I	17873	23395.27	1932.16	2461
3	I	19453	26310.15	2250.05	4833
4	I	22209	28863.56	2093.34	9123
5	I	24856	31428.24	2744.00	6207
6	I	26326	35175.77	3257.63	2999
7	I	29891	40296.96	4344.44	1596

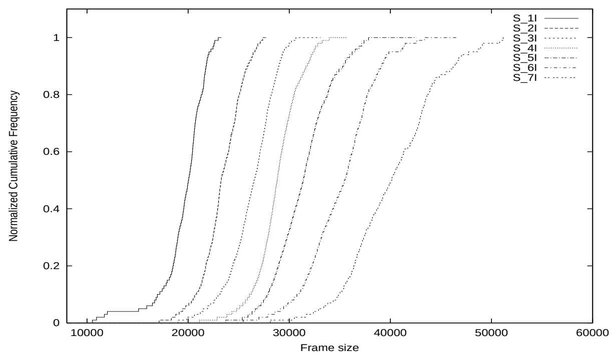

Fig. 1. Separation of I-Frames

C. Statistical Characterization of I-, P-, and B-frames

In this section we present the observations and analysis of data-sets obtained from our shot classification algorithm. Table I shows that geometrical separation of class boundaries kept enough data points in each class (smallest has

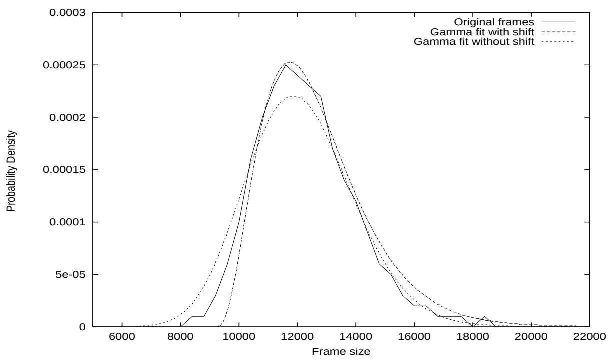

Fig. 2. Gamma fit for P Frames in Shot Class 5

1596) for doing meaningful statistical analysis and modeling. Before we discuss the statistical models, we present distributions of I-, P-, and B-frames of all seven classes.

Seven plots in Figure 1 show three facts: (*i*) The ordering created by our size-based classification of GOPs is preserved in the distribution of sizes of I frames. In other words, cumulative distribution of S_{kI} is quite regular and is to the left of that of $S_{(k+1)I}$, for all k, $1 \leq k \leq 6$. (*ii*) Frequency distributions of two sets of I-frames coming from two adjacent classes of GOPs have some overlap. (*iii*) Frequency distributions of two sets of I-frames coming from two non-adjacent classes of GOPs have very little or no overlap. These three characteristics of I-frame data-sets are also present in P- and B-frames data-sets (Figures not shown). Thus, geometrically separated GOP-size boundaries classified I-, P-, and B-frames well. A very useful observation is that in a class of small-size GOPs I-, P-, and B-frames are also small. It must be noted that such a regular statistical behavior may not hold for a GOP by GOP comparison.

Next task is to model distributions of I-, P-, and B-frames in each class. In other words, we need to find some standard statistical distributions to fit (3×7=) 21 empirical distributions. The observed patterns and quite successful frame-size models in [FNQ00] motivated us to model each frame-size distribution with a Gamma density function.

$$Gamma(x;\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{\frac{-x}{\beta}} \qquad (1)$$

Let $\overline{x}$ be the estimated mean and s^2 be the estimated variance of a data-set. If the data-set has been drawn from a Gamma distributed population, its parameters α and β can be estimated as $\hat{\beta} = s^2/\overline{x}$ and $\hat{\alpha} = \overline{x}/\hat{\beta}$.

Figure 2 shows that this conventional estimation of parameters shifts the distribution function to the left and height of the peak is smaller than that of the data-set. A close look and some educated guess led us to shift each point of the data-set by a constant number of bits, say u. (We discuss later how we choose the value of shift, u.) Estimates for two Gamma parameters, after shifting each point by u units, are given by $\hat{\beta} = s^2/(\overline{x} - u)$ and $\hat{\alpha} = (\overline{x} - u)/\hat{\beta}$. The selection of u, the shift of data, is complicated by the fact that in most data-sets there are a few points that are too far to the left or right. These data points should be excluded during analysis and modeling. There are many sophisticated techniques to identify these points. However, we resort to a simple heuristic: ignore one percent of the data points and set the value of the shift u at one percentile value. Table I shows shift-values, (mean, and standard deviation) for the seven data-sets corresponding to I-type frames. These distributions are used for generation of synthetic VBR videos. Results reported in Section IV demonstrate that they nicely capture statistical properties of the data they model.

The analysis technique used here for modeling duration of video segments is described next.

D. Formation of Video Segments

A video *segment* in a shot class is a *consecutive* sequence of clips belonging to that shot class. The *length* of a segment is the number of GOPs in the segment. We examined the clips in each shot class and formed all video segments and recorded their lengths. Following [DG99] a Gamma distribution with parameters $\hat{\alpha_{L_i}}$ and $\hat{\beta_{L_i}}$, estimated from observed length of segments in class S_i, is used for length of video segments in one of our video models. Next we present two methods for estimation of inter-class transitions of video-shot.

E. Inter-Shot Class Transitions

Let $P = |p_{ij}|, 1 \leq i, j \leq |S|$ denote a $|S| \times |S|$ transition probability matrix where p_{ij} gives the probability of transition from S_i to S_j as one traverses successive GOPs in G. We compute P by two different methods and call the resultant matrices P_A and P_B, respectively. The matrix P_A supports self-transitions but P_B excludes self -transitions. That is, the principal diagonal elements in P_B are zeros. The implications of these two different kind of transition probability matrices are discussed in section III.

The algorithms are quite similar and they both compute the transition probabilities from normalized relative frequency of transitions. For P_A, we set $p_{ij} = f_{ij}/f_i$, where f_{ij} is the number of transitions from S_i to S_j and f_i is the total number of transitions out of S_i. P_B is computed in a similar manner except that all self-transitions are ignored.

III. Modeling of Full Length Video

Two models for generation of video frame-size sequences are described next. They use Markov renewal processes; one uses only matrix P_A for inter- and intra-state transitions, and the other uses Gamma distributed random variables for lengths of video segments and P_B for inter-state transitions. In these models each shot class corresponds to one state of the underlying Markov chain.

A. Generation of Synthetic Video: Model A

In this model after all frames of a GOP are generated, the next state is determined by state transition matrix P_A. The process is repeated until the desired number of frames have been generated. This state transition method generates video segments whose lengths are geometrically distributed. The detailed algorithm follows:

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Algorithm Generate_Full_Video_Using_P_A
Input − n_f: *number of frames*
  $\hat{\alpha_{kR}}, \hat{\beta_{kR}}, u_{kR}; 1 \leq k \leq |S|, R \in \{I, B, P\}$
  P_A: *probability transition matrix*
Output − F': *a sequence of* n_f *frames*
begin
  s=initial random state; $1 \leq s \leq |S|$.
  $count = 0$; $F' = \phi$;
  while ($count < n_f$) {
    /**generate* **one** *GOP in state s**/
    $for(k = 1; k \leq N; k++)$ {
     /*N frames in a GOP*/
     $count = count + 1$;
     case GOP[k] :/**type of current frame**/
      I: draw $f \sim Gamma(\hat{\alpha_{sI}}, \hat{\beta_{sI}})$;
        $f = f + u_{sI}$;/**axis translation**/
      B: draw $f \sim Gamma(\hat{\alpha_{sB}}, \hat{\beta_{sB}})$;
        $f = f + u_{sB}$;
      P: draw $f \sim Gamma(\hat{\alpha_{sP}}, \hat{\beta_{sP}}$;
        $f = f + u_{sP}$;
     $insert < count, GOP(k), f >$ *into* F';
    }
    s=nextstate(s,P_A); /*change state using P_A*/
  }
end;

B. Generation of Synthetic Video: Model B

In this model, inter-state transitions are controlled by the state transition matrix P_B. The number of GOPs generated while in a state is explicitly modeled by a Gamma distribution whose parameters are estimated from the segment lengths of the original video. Thus, after a segment of video of a shot class has been generated, the transition matrix P_B is used to determine next shot class. The process is repeated until desired number of video frames have been generated. Details are omitted.

TABLE II
Percentage of Bit Loss in Original and Generated Movies

buf cap(c)	movie type	drain rate (d) 2.000	3.000	4.000	5.000
10 ms	original	14.222	2.360	0.183	0.002
	using P_A	14.410	2.414	0.193	0.006
	using P_B	13.650	2.010	0.129	0.003
20 ms	original	10.480	1.326	0.079	0.000
	using P_A	10.570	1.347	0.093	0.002
	using P_B	9.783	1.076	0.056	0.000

IV. Model Validation

We have presented multi-level characterization techniques for full-length VBR video data-sets. Also, two models for generation of synthetic full-length VBR video have been proposed. To validate these models, model-generated VBR videos have been compared with original VBR videos. Following standard techniques in the literature, we show QQ-plots, and data-loss observed from simulation of leaky-bucket.

TABLE III
Percentage of Frames Affected in Original Movie

buf cap(c)	frame type	drain rate (d) 2.000	3.000	4.000	5.000
10 ms	All	16.130	5.407	0.524	0.018
	I	92.770	32.053	3.146	0.107
	B	0.442	0.034	0.000	0.000
	P	2.241	0.251	0.000	0.000
20 ms	All	14.757	3.040	0.310	0.000
	I	85.637	18.031	1.860	0.000
	B	0.425	0.031	0.000	0.000
	P	1.210	0.083	0.000	0.000

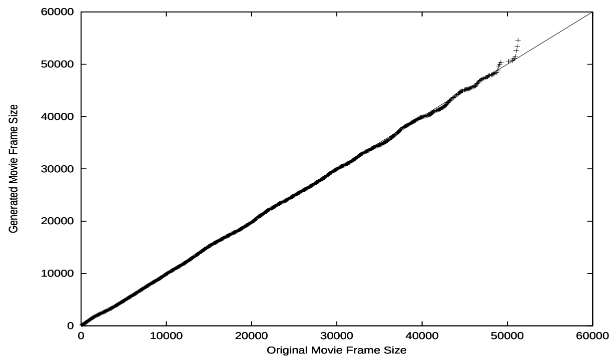

Fig. 3. QQ plot for the whole movie generated using P_A

A. QQ Plots

The *quartile-quartile* (QQ) plot of two data-sets is a visual inspection method for verification of their similarity. If two data-sets are identical, a straight line described by $y = x$, is obtained. Thus, closer the plot to the line $y = x$, better the similarity between the data-sets.

The plot in Figure 3 depicts similarity of original VBR video data-set with that synthetically generated using our Model A. As can be seen, frame sizes of two video are almost identical; only exceptions are a few large size frames in the synthetic movie. These exceptions may practically be ignored, since the number of frames are only a small fraction of a percent. Model B generated video produced similar QQ plot. Although Model A over-estimated the frame sizes and Model B under-estimated those, these deviations were very small and practically, frame-length distributions of synthetic and original videos are indistinguishable for both models.

B. Buffer Overflow Loss

For communication of VBR videos over B-ISDN, temporal ordering of the frames plays a critical role in DLR; for a given data transmission-rate, occurrence of long runs of large-size frames (*known as burstiness*) has higher DLR than the absence of them. Hence, temporal burstiness of original VBR video must be preserved in the data generated by a good model. Most commonly used test for measuring this behavior is passing the data through a generic buffer with capacity c, and drain rate d. For our study, buffer capacity is expressed in terms of mean frame-size of VBR source, and *independent of drain rate d*. For a 25-frame/s source, $c = 20$ ms corresponds to one half of a mean frame-size of the VBR video. The drain rate d is expressed in multiples of mean frame-size of VBR video.

Tables II, and III show data loss rate for various buffer capacities and drain rates. Each cell of Table II shows percentage data loss for original, and full-length synthetic movies from two models. For instance, when $c = 20$ ms and $d = 3$, the original movie suffers 1.326% data loss. With an identical buffer and transmission setting, full-length movies generated by Model A and Model B suffer 1.347% and 1.076% data loss. Although the differences are very small, one can see that (*i*) Model A shows a higher data loss than the original movie, and (*ii*) Model B shows a lower data loss than the original movie. This, observation also holds for other communication settings.

During our simulation study, percentage of total frames lost, and percentage of each type of frames lost were recorded. Some of the results are reported in Table III.[3] The additional insight obtained from the data is quite revealing: For the original movie, when $c = 20$ms and $d = 3$ **only 3.04%** of all frames are affected; a closer look reveals that most of them are I frames — **18.031%** I frames have been affected. The impact of an affected I frame propagates across the whole GOP containing the I frame. Thus, one can conclude that as high as 18.031% of the movie would be affected. To reduce percentage of affected I frames, one requires to increase buffer size (creating the effect of video smoothing), or the drain rate (increasing bandwidth allocation), or both.

V. Discussion and Conclusions

Frame-size based models of VBR videos, especially full-length movies, are essential for understanding the effect of data loss during transmission of MPEG-like compressed VBR video over B-ISDN. However, no satisfactory model has been reported in the past. The limited success of past efforts in frame-by-frame modeling of full-length VBR video is not because of lack of efforts, but because of the nature of complex nature of frame-size data-sets. Three different compression techniques applied to I-,P-, and B-frames produce different amount of compression. Moreover, different segments of a full-length movie produce frames of different sizes, because of composition or content of picture, and temporal similarity of adjacent pictures. A universal VBR video model must have enough parameters to capture all classes of video segments, and all three frame types (I, B, and P) in each class of video segments.

In this paper, we have presented algorithms for analysis of full-length VBR videos. After two-pass analysis of VBR video, our algorithms identify and partition (size-based) classes of video segments. Frames in each segment class produce three data-sets one each for I-, B-, and P-type frames. Each of these data-sets fits an *axis shifted* Gamma distribution, whose parameters are estimated form the data-set it models.

Using these Gamma distributions and Markov renewal processes we have proposed two models for generation of synthetic VBR videos. These, being *frame size* models, generate sizes of I-, B-, and P-frames. Thus, one can study types of frames being affected during communication.

We have used QQ plots to show visual similarity of model-generated VBR video data-sets with original data-set. Similarity of local burstiness of model-generated VBR videos and original video have been validated using leaky-bucket simulation technique. Full-length videos generated by both models preserved local burstiness of original video.

In summary, we provide not only two good models for generation of *synthetic* VBR video for study of B-ISDN, but also a tool for understanding of the quality of transmitted video when communication is subject to data loss. We are now analyzing more full-length videos for modeling them. Once a good number of videos have been modeled, we plan to study bandwidth gain obtainable from multiplexing them.

[3] To the best of our knowledge, no previous work has reported percentage of each type of frames lost.

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