

A Tableau Decision Procedure for Propositional Intuitionistic Logic

Alessandro Avellone, Guido Fiorino and Ugo Moscato

Dipartimento di Metodi Quantitativi per l'Economia, Università Milano-Bicocca
{alessandro.avellone,guido.fiorino,ugo.moscato}@unimib.it

Outline

- 1 Preliminaries**
 - Tableau calculus
 - Branching and Backtracking
 - Formulas groups
- 2 Optimizations**
 - Bounding depth: opt1
 - Bounding branching: opt2
 - Avoiding backtracking: opt3
- 3 PITP**
 - About the implementation
 - ILTP Library
- 4 Conclution and Future works**

Outline

- 1 Preliminaries**
 - Tableau calculus
 - Branching and Backtracking
 - Formulas groups
- 2 Optimizations
 - Bounding depth: opt1
 - Bounding branching: opt2
 - Avoiding backtracking: opt3
- 3 PITP
 - About the implementation
 - ILTP Library
- 4 Conclution and Future works

Calculus

Tableau calculus

- Enhancement of Fitting tableaux
- Related to tableau/sequent calculi: Dyckhoff, Hudelmaier, Miglioli, Moscato and Ornaghi
- Key words: Duplication free/contraction free, PSPACE-completeness

Tableau vs Kripke semantics

A tableau proof for a formula is the attempt to build a model satisfying the formula.

Calculus

Tableau calculus

- Enhancement of Fitting tableaux
- Related to tableau/sequent calculi: Dyckhoff, Hudelmaier, Miglioli, Moscato and Ornaghi
- Key words: Duplication free/contraction free, PSPACE-completeness

Tableau vs Kripke semantics

A tableau proof for a formula is the attempt to build a model satisfying the formula.

Rules

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Branching

The rules having more than one conclusion give rise to branches $\left(\frac{S, \mathbf{T}(A \vee B)}{S, \mathbf{TA} \mid S, \mathbf{TB}} \mathbf{T}\vee \right)$. Thus the search space consists of a proof whose branches have to be visited by the decision procedure.

Backtracking

In intuitionistic logic the order in which the rules are applied is relevant and affect the completeness. If the choice of a swff does not give a closed proof table, one has to backtrack and try with another swff (e.g. $\frac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{TA}, \mathbf{FB}} \mathbf{F} \rightarrow$).

Fact

The PSPACE-completeness of intuitionistic logic (Statman:79) suggests that backtracking cannot be eliminated.

Branching

The rules having more than one conclusion give rise to branches $\left(\frac{S, \mathbf{T}(A \vee B)}{S, \mathbf{TA} \mid S, \mathbf{TB}} \mathbf{T}\vee \right)$. Thus the search space consists of a proof whose branches have to be visited by the decision procedure.

Backtracking

In intuitionistic logic the order in which the rules are applied is relevant and affect the completeness. If the choice of a swff does not give a closed proof table, one has to backtrack and try with another swff (e.g. $\frac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{TA}, \mathbf{FB}} \mathbf{F} \rightarrow$).

Fact

The PSPACE-completeness of intuitionistic logic (Statman:79) suggests that backtracking cannot be eliminated.

Branching

The rules having more than one conclusion give rise to branches $\left(\frac{S, \mathbf{T}(A \vee B)}{S, \mathbf{TA} \mid S, \mathbf{TB}} \mathbf{T}\vee \right)$. Thus the search space consists of a proof whose branches have to be visited by the decision procedure.

Backtracking

In intuitionistic logic the order in which the rules are applied is relevant and affect the completeness. If the choice of a swff does not give a closed proof table, one has to backtrack and try with another swff (e.g. $\frac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{TA}, \mathbf{FB}} \mathbf{F} \rightarrow$).

Fact

The PSPACE-completeness of intuitionistic logic (Statman:79) suggests that backtracking cannot be eliminated.

Groups

Six group

The formulas are divided in six groups according to their behavior with respect to branching and backtracking

- $\mathcal{C}_1 = \{\mathbf{T}(A \wedge B), \mathbf{F}(A \vee B), \mathbf{F}_c(A \vee B), \mathbf{T}(\neg A), \mathbf{T}(p \rightarrow A) \text{ with } p \text{ an atom}, \mathbf{T}((A \wedge B) \rightarrow C), \mathbf{T}((A \vee B) \rightarrow C)\};$
- $\mathcal{C}_2 = \{\mathbf{T}(A \vee B), \mathbf{F}(A \wedge B)\};$
- $\mathcal{C}_3 = \{\mathbf{F}(\neg A), \mathbf{F}(A \rightarrow B)\};$
- $\mathcal{C}_4 = \{\mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(\neg A \rightarrow B)\};$
- $\mathcal{C}_5 = \{\mathbf{F}_c(A \rightarrow B), \mathbf{F}_c(\neg A)\};$
- $\mathcal{C}_6 = \{\mathbf{F}_c(A \wedge B)\}.$

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T_{\wedge} \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F_{\wedge} \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_{c\wedge}$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T_{\vee} \quad \frac{S, F(A \vee B)}{S, FA, FB} F_{\vee} \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_{c\vee}$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T_{\rightarrow} \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F_{\rightarrow} \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_{c\rightarrow}$$

$$\frac{S, T(\neg A)}{S, F_cA} T_{\neg} \quad \frac{S, F(\neg A)}{S_c, TA} F_{\neg} \quad \frac{S, F_c(\neg A)}{S_c, TA} F_{c\neg}$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T_{\rightarrow \wedge} \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T_{\rightarrow \neg}$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T_{\rightarrow \vee}$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T_{\rightarrow \rightarrow}$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Groups

$$\frac{S, T(A \wedge B)}{S, TA, TB} T\wedge \quad \frac{S, F(A \wedge B)}{S, FA|S, FB} F\wedge \quad \frac{S, F_c(A \wedge B)}{S_c, F_cA|S_c, F_cB} F_c\wedge$$

$$\frac{S, T(A \vee B)}{S, TA|S, TB} T\vee \quad \frac{S, F(A \vee B)}{S, FA, FB} F\vee \quad \frac{S, F_c(A \vee B)}{S, F_cA, F_cB} F_c\vee$$

$$\frac{S, TA, T(A \rightarrow B)}{S, TA, TB} T \rightarrow \text{Atom, with } A \text{ an atom}$$

$$\frac{S, F(A \rightarrow B)}{S_c, TA, FB} F \rightarrow \quad \frac{S, F_c(A \rightarrow B)}{S_c, TA, F_cB} F_c \rightarrow$$

$$\frac{S, T(\neg A)}{S, F_cA} T\neg \quad \frac{S, F(\neg A)}{S_c, TA} F\neg \quad \frac{S, F_c(\neg A)}{S_c, TA} F_c\neg$$

$$\frac{S, T((A \wedge B) \rightarrow C)}{S, T(A \rightarrow (B \rightarrow C))} T \rightarrow \wedge \quad \frac{S, T(\neg A \rightarrow B)}{S_c, TA|S, TB} T \rightarrow \neg$$

$$\frac{S, T((A \vee B) \rightarrow C)}{S, T(A \rightarrow p), T(B \rightarrow p), T(p \rightarrow C)} T \rightarrow \vee$$

$$\frac{S, T((A \rightarrow B) \rightarrow C)}{S_c, TA, Fp, T(p \rightarrow C), T(B \rightarrow p)|S, TC} T \rightarrow \rightarrow$$

where $S_c = \{TA|TA \in S\} \cup \{F_cA|F_cA \in S\}$ and p is a new atom

Outline

- 1 Preliminaries
 - Tableau calculus
 - Branching and Backtracking
 - Formulas groups
- 2 **Optimizations**
 - Bounding depth: opt1
 - Bounding branching: opt2
 - Avoiding backtracking: opt3
- 3 PITP
 - About the implementation
 - ILTP Library
- 4 Conclusion and Future works

Bounding depth: opt1

What happen if the CL-model underling S realizes S?

Example

$$S = \{T A, F B, T((A \rightarrow B) \rightarrow C)\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \triangleright S$$

$$\sigma \models A$$

$$\sigma \not\models B$$

$$\sigma \models (A \rightarrow B) \rightarrow C$$

$$\sigma \triangleright TA$$

$$\sigma \triangleright FB$$

$$\sigma \triangleright T((A \rightarrow B) \rightarrow C)$$

The Kripke model coinciding with the classical model σ realizes S .

Bounding depth: opt1

What happen if the CL-model underling S realizes S?

Example

$$S = \{\mathbf{T} A, \mathbf{F} B, \mathbf{T}((A \rightarrow B) \rightarrow C)\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \triangleright S$$

$$\sigma \models A$$

$$\sigma \not\models B$$

$$\sigma \models (A \rightarrow B) \rightarrow C$$

$$\sigma \triangleright \mathbf{T}A$$

$$\sigma \triangleright \mathbf{F}B$$

$$\sigma \triangleright \mathbf{T}((A \rightarrow B) \rightarrow C)$$

The Kripke model coinciding with the classical model σ realizes S.

we do not need to go on with the proof.

Bounding depth: opt1

What happen if the CL-model underling S realizes S?

Example

$$S = \{\mathbf{T} A, \mathbf{F} B, \mathbf{T}((A \rightarrow B) \rightarrow C)\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \triangleright S$$

$$\sigma \models A$$

$$\sigma \not\models B$$

$$\sigma \models (A \rightarrow B) \rightarrow C$$

$$\sigma \triangleright \mathbf{T}A$$

$$\sigma \triangleright \mathbf{F}B$$

$$\sigma \triangleright \mathbf{T}((A \rightarrow B) \rightarrow C)$$

The Kripke model coinciding with the classical model σ realizes S.

we do not need to go on with the proof.

Bounding depth: opt1

What happen if the CL-model underling S realizes S?

Example

$$S = \{\mathbf{T} A, \mathbf{F} B, \mathbf{T}((A \rightarrow B) \rightarrow C)\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \triangleright S$$

$$\sigma \models A$$

$$\sigma \not\models B$$

$$\sigma \models (A \rightarrow B) \rightarrow C$$

$$\sigma \triangleright \mathbf{T}A$$

$$\sigma \triangleright \mathbf{F}B$$

$$\sigma \triangleright \mathbf{T}((A \rightarrow B) \rightarrow C)$$

The Kripke model coinciding with the classical model σ realizes S.

we do not need to go on with the proof.

Bounding depth: opt1

and what happen if CL-model underling S does not realize S

Example

$$S = \{ \mathbf{T} A, \mathbf{F} B, \mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(A \rightarrow B) \}$$

$$\sigma = \{ A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false} \}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models \mathbf{T}(A \rightarrow B)$$

we **cannot** conclude
that S is closed

BUT...

Bounding depth: opt1

and what happen if CL-model underling S does not realize S

Example

$$S = \{T A, F B, T((A \rightarrow B) \rightarrow C), T(A \rightarrow B)\}$$

$$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models T(A \rightarrow B)$$

we **cannot** conclude
that S is closed

BUT...

Bounding depth: opt1

and what happen if CL-model underling S does not realize S

Example

$$S = \{T A, F B, T((A \rightarrow B) \rightarrow C), T(A \rightarrow B)\}$$

$$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models T(A \rightarrow B)$$

we **cannot** conclude
that S is closed

BUT...

Bounding depth: opt1

if every \mathcal{PV} in S is signed T or F_c

Example

$$S = \{T A, F_c B, T((A \rightarrow B) \rightarrow C), T(A \rightarrow B), F_c C\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models T(A \rightarrow B)$$

There is no Kripke
model realizing S

we do not need to
proceed with the proof.

Bounding depth: opt1

if every \mathcal{PV} in S is signed T or F_c

Example

$$S = \{T A, F_c B, T((A \rightarrow B) \rightarrow C), T(A \rightarrow B), F_c C\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models T(A \rightarrow B)$$

There is no Kripke
model realizing S

we do not need to
proceed with the proof.

Bounding depth: opt1

if every \mathcal{PV} in S is signed T or F_c

Example

$$S = \{T A, F_c B, T((A \rightarrow B) \rightarrow C), T(A \rightarrow B), F_c C\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \not\models S$$

$$\sigma \not\models A \rightarrow B \mid \sigma \not\models T(A \rightarrow B)$$

There is no Kripke
model realizing S

we do not need to
proceed with the proof.

Bounding branching: opt2

swffs whose intuitionistic truth coincides with classical truth

Example

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}(B \vee C), \mathbf{TA}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \not\models A \wedge B \quad \left| \quad \sigma \triangleright \mathbf{F}(A \wedge B)$$

$$\sigma \not\models A \wedge C \quad \left| \quad \sigma \triangleright \mathbf{F}(A \wedge C)$$

$$\sigma \not\models B \vee C \quad \left| \quad \sigma \not\triangleright \mathbf{T}(B \vee C)$$

we apply the rule related to $\mathbf{T}(B \vee C)$

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{TA}, \mathbf{TB}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{true}, \dots\}$$

$$\sigma \models A \wedge B \quad \left| \quad \sigma \not\triangleright \mathbf{F}(A \wedge B)$$

the set is contradictory.

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{TA}, \mathbf{TC}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{true}, \dots\}$$

$$\sigma \models A \wedge C \quad \left| \quad \sigma \not\triangleright \mathbf{F}(A \wedge C)$$

the set is contradictory.

swffs whose intuitionistic truth coincides with classical truth

Example

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}(B \vee C), \mathbf{TA}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{false}, C \rightarrow \text{false}\}$$

$$\sigma \not\models A \wedge B \quad \left| \quad \sigma \triangleright \mathbf{F}(A \wedge B)$$

$$\sigma \not\models A \wedge C \quad \left| \quad \sigma \triangleright \mathbf{F}(A \wedge C)$$

$$\sigma \not\models B \vee C \quad \left| \quad \sigma \not\triangleright \mathbf{T}(B \vee C)$$

we apply the rule related to $\mathbf{T}(B \vee C)$

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{TA}, \mathbf{TB}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{true}, \dots\}$$

$$\sigma \models A \wedge B \quad \left| \quad \sigma \not\triangleright \mathbf{F}(A \wedge B)$$

the set is contradictory.

$$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{TA}, \mathbf{TC}\}$$

$$\sigma = \{A \rightarrow \text{true}, B \rightarrow \text{true}, \dots\}$$

$$\sigma \models A \wedge C \quad \left| \quad \sigma \not\triangleright \mathbf{F}(A \wedge C)$$

the set is contradictory.

Avoiding backtracking: opt3

Permutations 1

Example

$$S = \{\mathbf{F}(A \rightarrow B), \mathbf{F}(C \rightarrow D)\}$$
$$\langle P, \leq, \rho, \Vdash \rangle, P = \{\rho\}, \Vdash = \emptyset.$$
 ρ

Avoiding backtracking: opt3

Permutations 1

Example

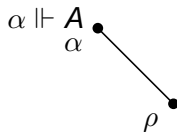
$$S = \{\mathbf{F}(A \rightarrow B), \mathbf{F}(C \rightarrow D)\}$$

$$\langle P, \leq, \rho, \Vdash \rangle, P = \{\rho\}, \Vdash = \emptyset.$$

$$S_1 = \{\mathbf{TA}, \mathbf{FB}\}$$

$$\langle P, \leq, \rho, \Vdash \rangle,$$

- $P = \{\rho, \alpha\},$
- $\Vdash = \{\alpha \Vdash A\}$



$$S_2 = \{\mathbf{TC}, \mathbf{FD}\}$$

$$\langle P, \leq, \rho, \Vdash \rangle,$$

- $P = \{\rho, \alpha, \beta\},$
- $\Vdash = \{\alpha \Vdash A, \beta \Vdash C\}$

Avoiding backtracking: opt3

Permutations 1

Example

$$S = \{ \mathbf{F}(A \rightarrow B), \mathbf{F}(C \rightarrow D) \}$$

$$\langle P, \leq, \rho, \Vdash \rangle, P = \{ \rho \}, \Vdash = \emptyset.$$

$$S_1 = \{ \mathbf{TA}, \mathbf{FB} \}$$

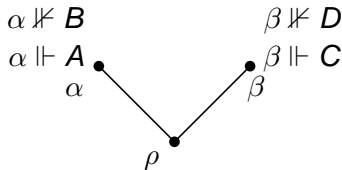
$$\langle P, \leq, \rho, \Vdash \rangle,$$

- $P = \{ \rho, \alpha \},$
- $\Vdash = \{ \alpha \Vdash A \}$

$$S_2 = \{ \mathbf{TC}, \mathbf{FD} \}$$

$$\langle P, \leq, \rho, \Vdash \rangle,$$

- $P = \{ \rho, \alpha, \beta \},$
- $\Vdash = \{ \alpha \Vdash A, \beta \Vdash C \}$

$$\rho$$


Avoiding backtracking: opt3

Permutations 2

Example

$$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$$

- $\tau(C) = A$,

- $\tau(D) = B$

$$\tau(S_2) = S_1$$

We can avoid backtracking on S .

Avoiding backtracking: opt3

Permutations 3

Example

$$S = \{ \begin{array}{l} T(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ T(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ T(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), F((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_1 = \{ \begin{array}{l} TP_0, FP_3, T(P_3 \rightarrow (P_1 \vee P_2)), T((P_1 \vee P_2) \rightarrow P_3), \\ T(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ T(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), F((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_2 = \{ \begin{array}{l} T(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ TP_2, FP_3, T(P_3 \rightarrow (P_0 \vee P_1)), T((P_0 \vee P_1) \rightarrow P_3), \\ T(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), F((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$$

$$\tau(P_0) = P_2,$$

$$\tau(P_1) = P_1$$

$$\tau(P_2) = P_0$$

$$\tau(P_3) = P_3$$

$$\tau(S_2) = S_1$$

Permutations 3

Example

$$S = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_1 = \{ \begin{array}{l} \mathbf{TP}_0, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_1 \vee P_2)), \mathbf{T}((P_1 \vee P_2) \rightarrow P_3), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_2 = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{TP}_2, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_0 \vee P_1)), \mathbf{T}((P_0 \vee P_1) \rightarrow P_3), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$$

$$\bullet \tau(P_0) = P_2,$$

$$\bullet \tau(P_1) = P_1$$

$$\bullet \tau(P_2) = P_0$$

$$\bullet \tau(P_3) = P_3$$

$$\tau(S_2) = S_1$$

Avoiding backtracking: opt3

Permutations 3

Example

$$S = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_1 = \{ \begin{array}{l} \mathbf{TP}_0, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_1 \vee P_2)), \mathbf{T}((P_1 \vee P_2) \rightarrow P_3), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_2 = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{TP}_2, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_0 \vee P_1)), \mathbf{T}((P_0 \vee P_1) \rightarrow P_3), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$$

$$\tau(P_0) = P_2,$$

$$\tau(P_1) = P_1$$

$$\tau(P_2) = P_0$$

$$\tau(P_3) = P_3$$

$$\tau(S_2) = S_1$$

Avoiding backtracking: opt3

Permutations 3

Example

$$S = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_1 = \{ \begin{array}{l} \mathbf{TP}_0, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_1 \vee P_2)), \mathbf{T}((P_1 \vee P_2) \rightarrow P_3), \\ \mathbf{T}(((P_2 \rightarrow (P_1 \vee P_0)) \rightarrow (P_1 \vee P_0))), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$S_2 = \{ \begin{array}{l} \mathbf{T}(((P_0 \rightarrow (P_1 \vee P_2)) \rightarrow (P_1 \vee P_2))), \\ \mathbf{TP}_2, \mathbf{FP}_3, \mathbf{T}(P_3 \rightarrow (P_0 \vee P_1)), \mathbf{T}((P_0 \vee P_1) \rightarrow P_3), \\ \mathbf{T}(((P_1 \rightarrow (P_2 \vee P_0)) \rightarrow (P_2 \vee P_0))), \mathbf{F}((P_1 \vee (P_2 \vee P_0))) \end{array} \}$$

$$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$$

$$\bullet \tau(P_0) = P_2,$$

$$\bullet \tau(P_1) = P_1$$

$$\bullet \tau(P_2) = P_0$$

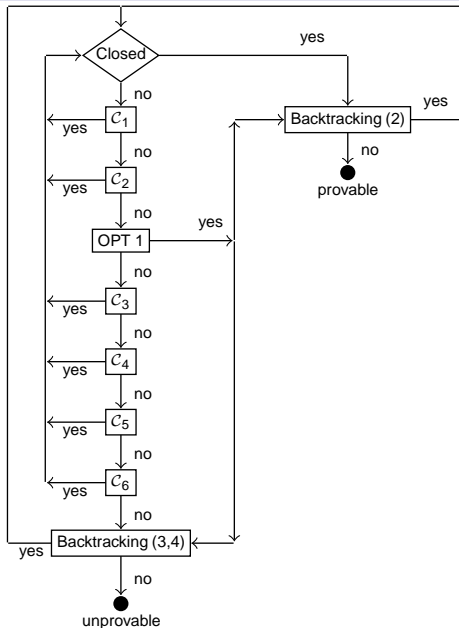
$$\bullet \tau(P_3) = P_3$$

$$\tau(S_2) = S_1$$

Outline

- 1 Preliminaries
 - Tableau calculus
 - Branching and Backtracking
 - Formulas groups
- 2 Optimizations
 - Bounding depth: opt1
 - Bounding branching: opt2
 - Avoiding backtracking: opt3
- 3 PITP**
 - About the implementation
 - ILTP Library
- 4 Conclusion and Future works

About the implementation



Remarks

- opt2 is executed during Group 2.
- opt3 is executed during Backtracking (3,4).
- In opt3, we search for τ such that $H = \tau(H')$ and $\tau = \tau^{-1}$.

ILTP Library (T. Raths, J. Otten, C. Kreitz.)

ILTP

- Contains 274 propositional problems; time limit: 600 sec., Xeon 3.4 GHz, Mandrake 10.2.
- 128 problems are solved-Theorems.
- 109 problems are solved-Non-Theorems.
- 37 problems are unsolved.
- Five provers:
 - ft Prolog: D. Sahlin, T. Franzen, S. Haridi (Swedish Institute of Computer Science),
 - ft C: D. Sahlin, T. Franzen, S. Haridi (Swedish Institute of Computer Science),
 - LJT: R. Dyckhoff (University of St Andrews),
 - STRIP: Dominique Larchey, Daniel Mery and Didier Galmiche (LORIA),
 - PITP: A. Avellone, G. Fiorino, U. Moscato (University of Milano-Bicocca).

ILTP Library (T. Raths, J. Otten, C. Kreitz.)

ILTP

- Three domains.
- LCL (2): Logic Calculi (TPTP).
- SYN (20): Syntactic problems have no obvious semantic interpretation (TPTP).
- SYJ (252): Intuitionistic syntactic problems have no obvious semantic interpretation.
 - SYJ201 (SYJ207): de Bruijn's ,
 - SYJ202 (SYJ208): Cook pigeon-hole,
 - SYJ203 (SYJ209): Formulae requiring many contractions,
 - SYJ204 (SYJ210): Formulae with normal natural deduction proofs only of exponential size,
 - SYJ205 (SYJ211): Formulae of Korn & Kreitz,
 - SYJ206 (SYJ212): Equivalences,

Result comparison 1

	ft Prolog	ft C	LJT	STRIP	PITP
solved	188	199	175	202	215
(%)	68.6	72.6	63.9	73.7	78.5
proved	104	106	108	119	128
refuted	84	93	67	83	87
solved after:					
0-1s	173	185	166	178	190
1-10s	5	6	4	11	10
10-100s	6	7	2	11	9
100-600s	4	1	3	2	6
(>600s)	86	75	47	43	58
errors	0	0	52	29	1

Result comparison 2

Provable

	SYJ202+1 provable	SYJ205+1 provable	SYJ206+1 provable
ft Prolog	07 (516.55)	08 (60.26)	10 (144.5)
ft C	07 (76.3)	09 (85.84)	11 (481.98)
LJT	02 (0.09)	20 (0.01)	05 (0.01)
STRIP	06 (11.28)	14 (267.39)	20 (37.64)
PITP	09 (595.79)	20 (0.01)	20 (4.07)

Refutable

	SYJ207+1 refutable	SYJ208+1 refutable	SYJ209+1 refutable	SYJ211+1 refutable	SYJ212+1 refutable
ft Prolog	07 (358.05)	08 (65.41)	10 (543.09)	04 (66.62)	20 (0.01)
ft C	07 (51.13)	17 (81.41)	10 (96.99)	04 (17.25)	20 (0.01)
LJT	03 (2.64)	08 (0.18)	10 (461.27)	08 (546.46)	07 (204.98)
STRIP	04 (9.3)	06 (0.24)	10 (132.55)	09 (97.63)	20 (36.79)
PITP	04 (11.11)	08 (83.66)	10 (280.47)	20 (526.16)	11 (528.08)

Result comparison 2

Provable

	SYJ201+1	SYJ202+1
PITP none	20 (1.29)	03 (0.01)
PITP -opt1	20 (0.03)	08 (44.59)
PITP -opt2	20 (1.67)	03 (0.01)
PITP -opt3	20 (0.03)	08 (44.21)
PITP ALL	20 (0.03)	08 (45.30)

Refutable

	SYJ207+1	SYJ208+1	SYJ209+1	SYJ211+1	SYJ212+1
PITP none	04 (43.77)	04 (2.50)	10 (596.55)	20 (526.94)	11 (527.72)
PITP -opt1	04 (44.76)	08 (93.60)	10 (325.93)	20 (558.11)	11 (548.01)
PITP -opt2	04 (12.18)	04 (2.37)	10 (311.37)	19 (293.34)	10 (88.92)
PITP -opt3	04 (11.36)	08 (94.30)	10 (591.68)	19 (291.18)	10 (92.05)
PITP ALL	04 (12.74)	08 (90.11)	10 (297.83)	19 (313.11)	10 (93.18)

Outline

- 1 Preliminaries
 - Tableau calculus
 - Branching and Backtracking
 - Formulas groups
- 2 Optimizations
 - Bounding depth: opt1
 - Bounding branching: opt2
 - Avoiding backtracking: opt3
- 3 PITP
 - About the implementation
 - ILTP Library
- 4 **Conclution and Future works**

Future works

- Improve permutations.
- New optimization: 234 problems are solved (85,4%).
- Implement a parallel version of the prover.