Midterm Rubric and Selected Answers

Monday, 23 March 2024
10:10–11:00 AM

There are 4 problems each worth 6 points. No notes, no collaboration. Please respect and uphold the integrity of the examination process. Sign the cover page to show agreement with these instructions.

Name: _____

Problem	Credit
1	
2	
3	
4	
Total	

1. Write a context free grammar for the language,

$$\{\, a^i\, b^j\, a^{i+j} \mid i, j \geq 0 \,\}$$

Answer:

$$
\begin{aligned}
S &\; \rightarrow \; aSa \mid T \\
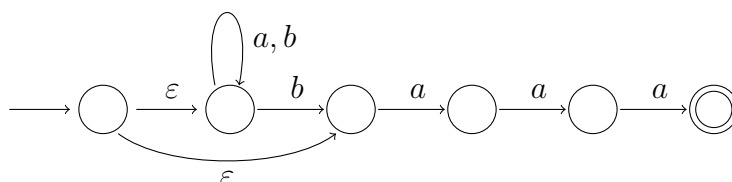T &\; \rightarrow \; bTa \mid \varepsilon
\end{aligned}
$$

Rubric:

points	requirements
6	all correct
5	string ba not recognized or off by one.
3	the quantities correct but not order.
2	a grammar was written
0	there was no CFF

2. Give an NFA that accepts the language over $\{\,a, b\,\}^*$ for string that end in *exactly* three a's.

For instance, aaa and $baaa$ are in the language, but $aaaa$ is not.

Answer:

Rubric:

points	requirements
6	all correct
5	one error from correct
4	two errors from correct
3	a wrong solution but that handles some cases
2	a wrong solution that handles only a few cases
0	diagram deficient

At popular wrong response is a four state DFA leading forward on a's with the last state final; each state has a transition on b to the start state, and final state also as a transition on a to the start state. This DFA accepts $aaa(aaaa)^*$. Variations which were considered equivalent left out arrows leaving the final state.

Responses that were 2 points generally how only the states leading forward, so the language is b^*aaa. Or if the presence of the substring aab disqualified the string.
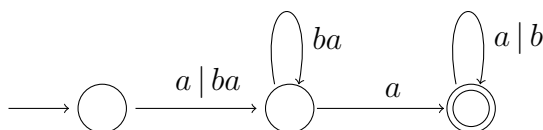
3. Give a Regular Expression for strings in $\{\,a,b\,\}^*$ of the following form:

    (a) the string must contain the substring aa and

    (b) all substrings bb (if any) must occur after the first aa.

For instance, the strings $aabb$ and $baabbabb$ are in the language. The strings $bbaa$ and aba are not in the language.

Hint: Sketch a DFA.

Answer:

$$(a\,|\,ba)(ba)^*a\,(a\,|\,b)^*$$

another answer,

$$((ab)^*\,|\,b(ab)^*)\,aa\,(a\,|\,b)^*$$

Rubric:

Perfect responses were verified. Expressions that could be easily corrected were corrected at one point per correction.

The wrong answers were scored on their difference from correct by the number of errors made over exhaustive testing on all strings of length 11.

points	number wrong
6	regular expression correct
5	regular expression correct with one correction
4	
3	from 40 to 100
2	from 100 to 1000
1	more than 1000
0	could not be scored

4. Write a context free grammar for the language of all strings in $\{a, b\}^*$ that are *not* of the form $a^i b^i$. That is, the complement language of $\{a^i b^i \mid i \geq 0\}$.

   Hint: When are strings not of the form $a^i b^i$?

   Answer:

   Rubric:

   Difficult programs were entered into a CFG program and run. An improved answer included exactly the language $a^i b^j$ for $i \neq j$. Other answers were analyzed, and checked if $a^i b^i$ was rejected and $a^i b^j$ for $j \neq j$ accepted.

points	requirements
6	correct
4	improved answer
2	most answers
0	nothing gradable