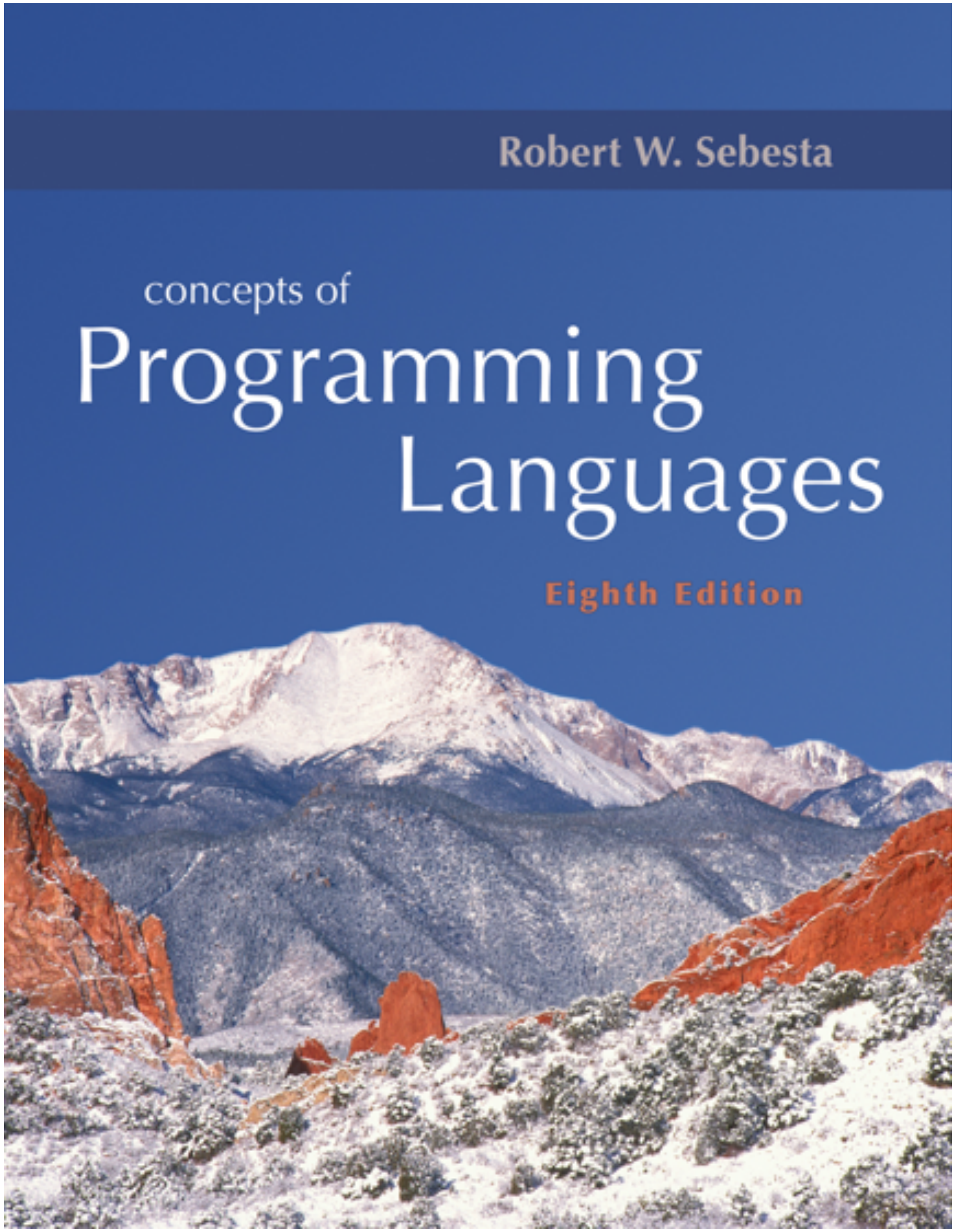
Chapter 15

Functional Programming Languages



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Chapter 15 Topics

- Introduction
- Mathematical Functions
- Fundamentals of Functional Programming Languages
- The First Functional Programming Language: LISP
- Introduction to Scheme
- COMMON LISP
- ML
- Haskell
- Applications of Functional Languages
- Comparison of Functional and Imperative Languages

Introduction

- The design of the imperative languages is based directly on the von Neumann architecture
  - Efficiency is the primary concern, rather than the suitability of the language for software development
- The design of the functional languages is based on mathematical functions
  - A solid theoretical basis that is also closer to the user, but relatively unconcerned with the architecture of the machines on which programs will run

Mathematical Functions

- A mathematical function is a mapping of members of one set, called the domain set, to another set, called the range set
- A lambda expression specifies the parameter(s) and the mapping of a function in the following form

`λ(x) x * x * x`

for the function `cube (x) ≡ x * x * x`

where x is a real number

Lambda Expressions

- Lambda expressions describe nameless functions
- Lambda expressions are applied to parameter(s) by placing the parameter(s) after the expression

e.g., `(λ(x) x * x * x)(2)`

which evaluates to `8`

Functional Forms

- A higher-order function, or functional form, is one that either takes functions as parameters or yields a function as its result, or both
- Common forms are function composition and apply-to-all

Function Composition

- A functional form that takes two functions as parameters and yields a function whose value is the first actual parameter function applied to the application of the second

Form: `h ≡ f ° g`

which means `h (x) ≡ f (g(x))`

For `f (x) ≡ x + 2` and `g (x) ≡ 3 * x`,

`h ≡ f ° g` yields `(3 * x) + 2`

Apply-to-all

- A functional form that takes a single function as a parameter and yields a list of values obtained by applying the given function to each element of a list of parameters

Form: α

For `h (x) ≡ x * x`

`α( h, (2, 3, 4))` yields `(4, 9, 16)`

Fundamentals of Functional Programming Languages

- The objective of the design of a FPL is to mimic mathematical functions to the greatest extent possible
- The basic process of computation is fundamentally different in a FPL than in an imperative language
  - In an imperative language, operations are done and the results are stored in variables for later use
  - Management of variables is a constant concern and source of complexity for imperative programming
- In an FPL, variables are not necessary, as is the

Referential Transparency

- In an FPL, the evaluation of a function always produces the same result given the same parameters

LISP Data Types and Structures

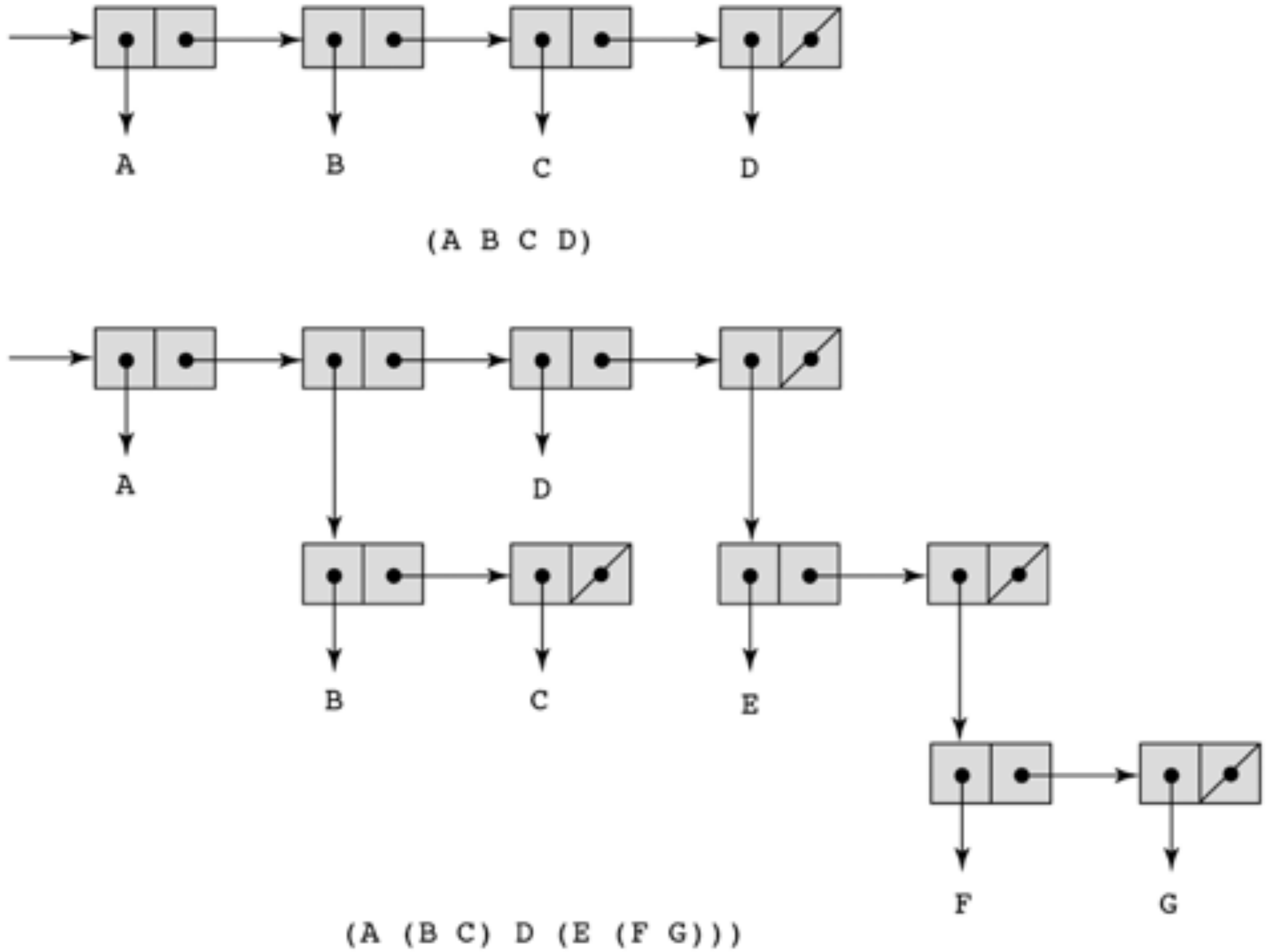
- Data object types: originally only atoms and lists
- List form: parenthesized collections of sublists and/or atoms

  e.g., `(A B (C D) E)`

  - Originally, LISP was a typeless language
  - LISP lists are stored internally as single-linked lists

Figure 15.1

Internal representation of two LISP lists



LISP Interpretation

- Lambda notation is used to specify functions and function definitions. Function applications and data have the same form.

e.g., If the list `(A B C)` is interpreted as data it is

a simple list of three atoms, `A`, `B`, and `C`

If it is interpreted as a function application,

it means that the function named `A` is

applied to the two parameters, `B` and `C`

- The first LISP interpreter appeared only as a demonstration of the universality of the computational capabilities of the notation

Origins of Scheme

- A mid-1970s dialect of LISP, designed to be a cleaner, more modern, and simpler version than the contemporary dialects of LISP
- Uses only static scoping
- Functions are first-class entities
  - They can be the values of expressions and elements of lists
  - They can be assigned to variables and passed as parameters

Evaluation

- Parameters are evaluated, in no particular order
- The values of the parameters are substituted into the function body
- The function body is evaluated
- The value of the last expression in the body is the value of the function

Primitive Numeric Functions

- Arithmetic: `+, -, *, /, ABS, SQRT, REMAINDER, MIN, MAX`

  e.g., `(+ 5 2)` yields `7`

- `QUOTE` – takes one parameter; returns the parameter without evaluation
  - `QUOTE` is required because the Scheme interpreter, named `EVAL`, always evaluates parameters to function applications before applying the function. `QUOTE` is used to avoid parameter evaluation when it is not appropriate
  - `QUOTE` can be abbreviated with the apostrophe prefix operator

    `'(A B)` is equivalent to `(QUOTE (A B))`

Function Definition: `LAMBDA`

- Lambda Expressions
  - Form is based on λ notation

e.g., `(LAMBDA (x) (* x x))`

`x` is called a bound variable

- Lambda expressions can be applied

e.g., `((LAMBDA (x) (* x x)) 7)`

Special Form Function: `DEFINE`

- A Function for Constructing Functions DEFINE – Two forms:
  - To bind a symbol to an expression

    e.g., `(DEFINE pi 3.141593)`

    Example use: `(DEFINE two_pi (* 2 pi))`
  - To bind names to lambda expressions

    e.g., `(DEFINE (square x) (* x x))`

    Example use: `(square 5)`

- The evaluation process for `DEFINE` is different! The first parameter is never evaluated. The second parameter is evaluated and bound to the first parameter.

Output Functions

- `(DISPLAY expression)`
- `(NEWLINE)`

Numeric Predicate Functions

- `#T` is true and `#F` is false (sometimes `()` is used for false)
- `=, <>, >, <, >=, <=`
- `EVEN?, ODD?, ZERO?, NEGATIVE?`

Control Flow: `IF`

- Selection– the special form, `IF`

```
(IF predicate then_exp else_exp)
```

e.g.,

```
(IF (<> count 0)
   (/ sum count)
   0)
```

Control Flow: COND

- Multiple Selection – the special form, COND

General form:

```
(COND
    (predicate_1  expr  {expr})
    (predicate_2  expr  {expr})
    . . .
    (predicate_n  expr  {expr})
    [(ELSE  expr  {expr})]
)
```

- Returns the value of the last expression in the first pair whose predicate evaluates to true

Example of COND

```
(DEFINE (compare x y)
   (COND
     ((> x y) “x is greater than y”)
     ((< x y) “y is greater than x”)
     (ELSE “x and y are equal”)
   )
 )
```

List Functions: `CAR` and `CDR`

- `CAR` takes a list parameter; returns the first element of that list

  e.g., `(CAR '(A B C))` yields `A`

  `(CAR '((A B) C D))` yields `(A B)`

- `CDR` takes a list parameter; returns the list after removing its first element

  e.g., `(CDR '(A B C))` yields `(B C)`

  `(CDR '((A B) C D))` yields `(C D)`

List Functions: `CONS` and `LIST`

- `CONS` takes two parameters, the first of which can be either an atom or a list and the second of which is a list; returns a new list that includes the first parameter as its first element and the second parameter as the remainder of its result

  e.g., `(CONS 'A '(B C))` returns `(A B C)`

- `LIST` takes any number of parameters; returns a list with the parameters as elements

Predicate Function: `EQ?`

- `EQ?` takes two symbolic parameters; it returns `#T` if both parameters are atoms and the two are the same

  e.g., `(EQ? 'A 'A)` yields `#T`

  `(EQ? 'A 'B)` yields `#F`

  - Note that if `EQ?` is called with list parameters, the result is not reliable
  - Also `EQ?` does not work for numeric atoms

Predicate Functions: `LIST?` and `NULL?`

- `LIST?` takes one parameter; it returns `#T` if the parameter is a list; otherwise `#F`
- `NULL?` takes one parameter; it returns `#T` if the parameter is the empty list; otherwise `#F`
  - Note that `NULL?` returns `#T` if the parameter is `()`

Example Scheme Function: `member`

- `member` takes an atom and a simple list; returns `#T` if the atom is in the list; `#F` otherwise

```
(DEFINE (member atm lis)
(COND
    ((NULL? lis) #F)
    ((EQ? atm (CAR lis)) #T)
    ((ELSE (member atm (CDR lis)))
))
```

Example Scheme Function: `equalsimp`

- `equalsimp` takes two simple lists as parameters; returns `#T` if the two simple lists are equal; `#F` otherwise

```
 (DEFINE (equalsimp lis1 lis2)
 (COND
    ((NULL? lis1) (NULL? lis2))
    ((NULL? lis2) #F)
    ((EQ? (CAR lis1) (CAR lis2))
         (equalsimp(CDR lis1)(CDR lis2)))
    (ELSE #F)
))
```

Example Scheme Function: `equal`

- `equal` takes two general lists as parameters; returns `#T` if the two lists are equal; `#F` otherwise

```
(DEFINE (equal lis1 lis2)
   (COND
     ((NOT (LIST? lis1))(EQ? lis1 lis2))
     ((NOT (LIST? lis2)) #F)
     ((NULL? lis1) (NULL? lis2))
     ((NULL? lis2) #F)
     ((equal (CAR lis1) (CAR lis2))
           (equal (CDR lis1) (CDR lis2)))
     (ELSE #F)
))
```

Example Scheme Function: `append`

- `append` takes two lists as parameters; returns the first parameter list with the elements of the second parameter list appended at the end

```
(DEFINE (append lis1 lis2)
   (COND
     ((NULL? lis1) lis2)
     (ELSE (CONS (CAR lis1)
                       (append (CDR lis1) lis2)))
))
```

Example Scheme Function: `LET`

- General form:

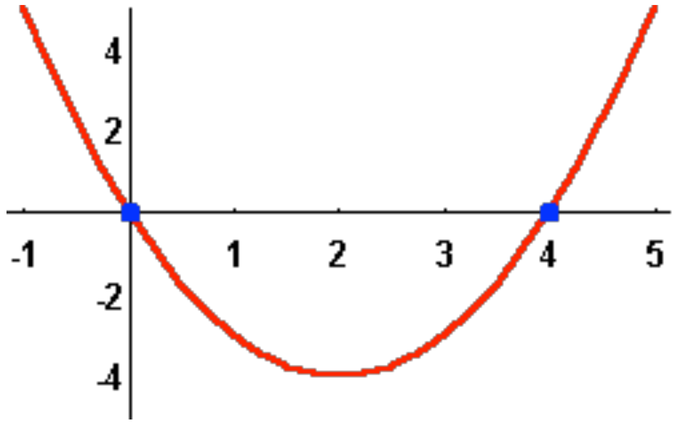
```
(LET (
    (name_1 expression_1)
    (name_2 expression_2)
    ...
    (name_n expression_n))
   body
)
```

- Evaluate all expressions, then bind the values to the names; evaluate the body

LET Example

```
(DEFINE (quadratic_roots a b c)
   (LET
     (
      (root_part_over_2a (/ (SQRT (- (* b b) (* 4 a c)))(* 2 a)))
      (minus_b_over_2a   (/ (- 0 b) (* 2 a)))
     )
    (LIST (+ minus_b_over_2a root_part_over_2a)
          (- minus_b_over_2a root_part_over_2a))
   )
)
```

$$ax^2 + bx + c = 0$$



Scheme Functional Forms

- Composition
  - The previous examples have used it
  - `(CDR  (CDR '(A B C)))` returns `(C)`
- Apply to All – one form in Scheme is `mapcar`
  - Applies the given function to all elements of the given list;

```
(DEFINE (mapcar fun lis)
  (COND
    ((NULL? lis) ())
    (ELSE (CONS (fun (CAR lis))
                (mapcar fun (CDR lis))))
))
```

Functions That Build Code

- It is possible in Scheme to define a function that builds Scheme code and requests its interpretation
- This is possible because the interpreter is a user-available function, `EVAL`

Adding a List of Numbers

```
((DEFINE (adder lis)
   (COND
     ((NULL? lis) 0)
     (ELSE (EVAL (CONS '+ lis)))
))
```

- The parameter is a list of numbers to be added; `adder` inserts a + operator and evaluates the resulting list
  - Use `CONS` to insert the atom + into the list of numbers.
  - Be sure that + is quoted to prevent evaluation
  - Submit the new list to `EVAL` for evaluation

COMMON LISP

- A combination of many of the features of the popular dialects of LISP around in the early 1980s
- A large and complex language--the opposite of Scheme
- Features include:
  - records
  - arrays
  - complex numbers
  - character strings
  - powerful I/O capabilities
  - packages with access control
  - iterative control statements

ML

- A static-scoped functional language with syntax that is closer to Pascal than to LISP
- Uses type declarations, but also does type inferencing to determine the types of undeclared variables
- It is strongly typed (whereas Scheme is essentially typeless) and has no type coercions
- Includes exception handling and a module facility for implementing abstract data types
- Includes lists and list operations

ML Specifics

- Function declaration form:

  `fun` name (parameters) = body;

  e.g., `fun cube (x : int) = x * x * x;`

- The type could be attached to return value, as in

  `fun cube (x) : int = x * x * x;`

  With no type specified, it would default to int (the default for numeric values)

- User-defined overloaded functions are not allowed, so if we wanted a `cube` function for real parameters, it would need to have a different name

- There are no type coercions in ML

ML Specifics (continued)

- ML selection

  `if` expression `then` then_expression

  `else` else_expression

where the first expression must evaluate to a Boolean value

- Pattern matching is used to allow a function to operate on different parameter forms

```
fun fact(0) = 1
| fact(n : int) : int = n * fact(n - 1)
```

ML Specifics (continued)

- Lists

  Literal lists are specified in brackets

  `[3, 5, 7]`

  `[]` is the empty list

  `CONS` is the binary infix operator, `::`

  `4 :: [3, 5, 7]`, which evaluates to `[4, 3, 5, 7]`

  `CAR` is the unary operator `hd`

  `CDR` is the unary operator `tl`

```
fun length([]) = 0
|   length(h :: t) = 1 + length(t);

fun append([], lis2) = lis2
|   append(h :: t, lis2) = h :: append(t, lis2);
```

ML Specifics (continued)

- The `val` statement binds a name to a value (similar to `DEFINE` in Scheme)

```
val distance = time * speed;
```

  - As is the case with `DEFINE`, `val` is nothing like an assignment statement in an imperative language

Haskell

- Similar to ML (syntax, static scoped, strongly typed, type inferencing, pattern matching)
- Different from ML (and most other functional languages) in that it is purely functional (e.g., no variables, no assignment statements, and no side effects of any kind)

Syntax differences from ML (no reserved word for functions, parenthesis optional, alternative definitions of functions have same appearance)

```
fact 0 = 1
fact n = n * fact (n - 1)

fib 0 = 1
fib 1 = 1
fib (n + 2) = fib (n + 1) + fib n
```

Function Definitions with Different Parameter Ranges

```
fact n
    |  n == 0 = 1
    |  n > 0 = n * fact(n - 1)


  sub n
     | n < 10 = 0
     | n > 100       = 2
     | otherwise     = 1


  square x = x * x
```

– Works for any numeric type of `x`

Lists

- List notation: Put elements in brackets

  e.g., `directions = ["north","south", "east","west"]`
- Length: `#`

  e.g., `#directions` is `4`
- Arithmetic series with the `..` operator

  e.g., `[2, 4..10]` is `[2, 4, 6, 8, 10]`
- Catenation is with `++`

  e.g., `[1, 3] ++ [5, 7]` results in `[1, 3, 5, 7]`

- `CONS`, `CAR`, `CDR` via the colon operator (as in Prolog)

  e.g., `1:[3, 5, 7]` results in `[1, 3, 5, 7]`

Factorial Revisited

```
product [] = 1
product (a:x) = a * product x

fact n = product [1..n]
```

List Comprehension

- Set notation
- List of the squares of the first 20 positive integers: `[n * n | n ← [1..20]]`
- All of the factors of its given parameter:

```
factors n = [i | i ← [1..n `div` 2], n `mod` i == 0]
```

Quicksort

```
sort [] = []
sort (a:x) =
      sort [b | b ← x; b <= a] ++
      [a] ++
      sort [b | b ← x; b > a]
```

Example: one-liner

```
let qsOneL xs = concat [qsOneL [y | y <- tail xs, y < x] ++ x : qsOneL [y | y <- tail xs, y >= x] | x <- take 1 xs]
```

```pascal
 * A Pascal quicksort.
 *****************************************************************************}
PROGRAM Sort(input, output);
    CONST
        { Max array size. }
        MaxElts = 50;
    TYPE
        { Type of the element array. }
        IntArrType = ARRAY [1..MaxElts] OF Integer;

    VAR
        { Indexes, exchange temp, array size. }
        i, j, tmp, size: integer;

        { Array of ints }
        arr: IntArrType;

    { Read in the integers. }
    PROCEDURE ReadArr(VAR size: Integer; VAR a: IntArrType);
        BEGIN
            size := 1;
            WHILE NOT eof DO BEGIN
                readln(a[size]);
                IF NOT eof THEN
                    size := size + 1
            END
        END;

           PROCEDURE QuicksortRecur(start, stop: integer);
            VAR
                m: integer;

                { The location separating the high and low parts. }
                splitpt: integer;

            { The quicksort split algorithm.  Takes the range, and
              returns the split point. }
            FUNCTION Split(start, stop: integer): integer;
                VAR
                    left, right: integer;       { Scan pointers. }
                    pivot: integer;             { Pivot value. }

                { Interchange the parameters. }
                PROCEDURE swap(VAR a, b: integer);
                    VAR
                        t: integer;
                    BEGIN
                        t := a;
                        a := b;
                        b := t
                    END;

                BEGIN { Split }
                    { Set up the pointers for the hight and low sections, and
                      get the pivot value. }
                    pivot := arr[start];
                    left := start + 1;
                    right := stop;

                    { Look for pairs out of place and swap 'em. }
                    WHILE left <= right DO BEGIN
                        WHILE (left <= stop) AND (arr[left] < pivot) DO
                            left := left + 1;
                        WHILE (right > start) AND (arr[right] >= pivot) DO
                            right := right - 1;
                        IF left < right THEN
                            swap(arr[left], arr[right]);
                    END;

                    { Put the pivot between the halves. }
                    swap(arr[start], arr[right]);

                    { This is how you return function values in pascal.
                      Yeccch. }
                    Split := right
                END;

            BEGIN { QuicksortRecur }
                { If there's anything to do... }
                IF start < stop THEN BEGIN
                    splitpt := Split(start, stop);
                    QuicksortRecur(start, splitpt-1);
                    QuicksortRecur(splitpt+1, stop);
                END
            END;

        BEGIN { Quicksort }
            QuicksortRecur(1, size)
        END;

    BEGIN
        { Read }
        ReadArr(size, arr);

        { Sort the contents. }
        Quicksort(size, arr);

        { Print. }
        FOR i := 1 TO size DO
            writeln(arr[i])
    END.
```

Source: http://sandbox.mc.edu/~bennet/cs404/doc/qsort_pas.html

```java
   /***********************************************************************
    *  Quicksort code from Sedgewick 7.1, 7.2.
    ***********************************************************************/
    public static void quicksort(double[] a) {
        shuffle(a);                            // to guard against worst-case
        quicksort(a, 0, a.length - 1);
    }

    // quicksort a[left] to a[right]
    public static void quicksort(double[] a, int left, int right) {
        if (right <= left) return;
        int i = partition(a, left, right);
        quicksort(a, left, i-1);
        quicksort(a, i+1, right);
    }

    // partition a[left] to a[right], assumes left < right
    private static int partition(double[] a, int left, int right) {
        int i = left - 1;
        int j = right;
        while (true) {
            while (less(a[++i], a[right]))      // find item on left to swap
                ;                               // a[right] acts as sentinel
            while (less(a[right], a[--j]))      // find item on right to swap
                if (j == left) break;           // don't go out-of-bounds
            if (i >= j) break;                  // check if pointers cross
            exch(a, i, j);                      // swap two elements into place
        }
        exch(a, i, right);                      // swap with partition element
        return i;
    }

    // is x < y ?
    private static boolean less(double x, double y) {
        comparisons++;
        return (x < y);
    }

    // exchange a[i] and a[j]
    private static void exch(double[] a, int i, int j) {
        exchanges++;
        double swap = a[i];
        a[i] = a[j];
        a[j] = swap;
    }

    // shuffle the array a[]
    private static void shuffle(double[] a) {
        int N = a.length;
        for (int i = 0; i < N; i++) {
            int r = i + (int) (Math.random() * (N-i));   // between i and N-1
            exch(a, i, r);
        }
    }



    // test client
    public static void main(String[] args) {
        int N = Integer.parseInt(args[0]);

        // generate N random real numbers between 0 and 1
        long start = System.currentTimeMillis();
        double[] a = new double[N];
        for (int i = 0; i < N; i++)
            a[i] = Math.random();
        long stop = System.currentTimeMillis();
        double elapsed = (stop - start) / 1000.0;
        System.out.println("Generating input:  " + elapsed + " seconds");

        // sort them
        start = System.currentTimeMillis();
        quicksort(a);
        stop = System.currentTimeMillis();
        elapsed = (stop - start) / 1000.0;
        System.out.println("Quicksort:   " + elapsed + " seconds");

        // print statistics
        System.out.println("Comparisons: " + comparisons);
        System.out.println("Exchanges:   " + exchanges);
    }
}
```

Source: http://www.cs.princeton.edu/introcs/42sort/QuickSort.java.html

Lazy Evaluation

- A language is strict if it requires all actual parameters to be fully evaluated
- A language is nonstrict if it does not have the strict requirement
- Nonstrict languages are more efficient and allow some interesting capabilities – infinite lists
- Lazy evaluation – Only compute those values that are necessary
- Positive numbers

```
positives = [0..], evens = [2,4,..]
squares = [n * n | n ← [0..]]
```

- Determining if `16` is a square number

```
member squares 16
```

Member Revisited

- The member function could be written as:

```
member [] b = False
member(a:x) b=(a == b)||member x b
```

- However, this would only work if the parameter to squares was a perfect square; if not, it will keep generating them forever. The following version will always work:

```
member2 (m:x) n
    | m < n = member2 x n
    | m == n = True
    | otherwise = False
```

Applications of Functional Languages

- APL is used for throw-away programs
- LISP is used for artificial intelligence
  - Knowledge representation
  - Machine learning
  - Natural language processing
  - Modeling of speech and vision
- Scheme is used to teach introductory programming at some universities

Comparing Functional and Imperative Languages

- Imperative Languages:
  - Efficient execution
  - Complex semantics
  - Complex syntax
  - Concurrency is programmer designed
- Functional Languages:
  - Simple semantics
  - Simple syntax
  - Inefficient execution
  - Programs can automatically be made

Summary

- Functional programming languages use function application, conditional expressions, recursion, and functional forms to control program execution instead of imperative features such as variables and assignments
- LISP began as a purely functional language and later included imperative features
- Scheme is a relatively simple dialect of LISP that uses static scoping exclusively
- COMMON LISP is a large LISP-based language
- ML is a static-scoped and strongly typed functional language which includes type inference, exception handling, and a variety of data structures and abstract data types
- Haskell is a lazy functional language supporting infinite lists and set comprehension.
- Purely functional languages have advantages over imperative alternatives, but their lower efficiency on existing machine architectures has prevented them from enjoying widespread use