# Reinforcement Learning

Odelia Schwartz
2019

# Forms of learning?

# Forms of learning

- Unsupervised learning
- Supervised learning
- Reinforcement learning

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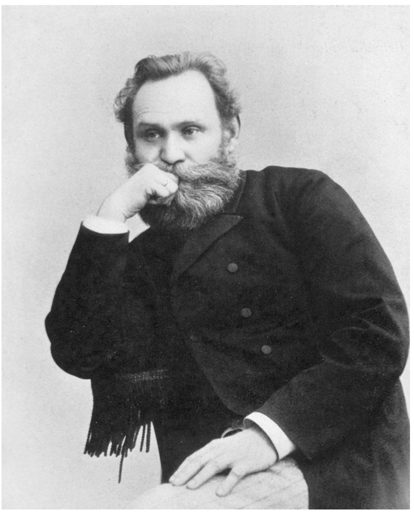
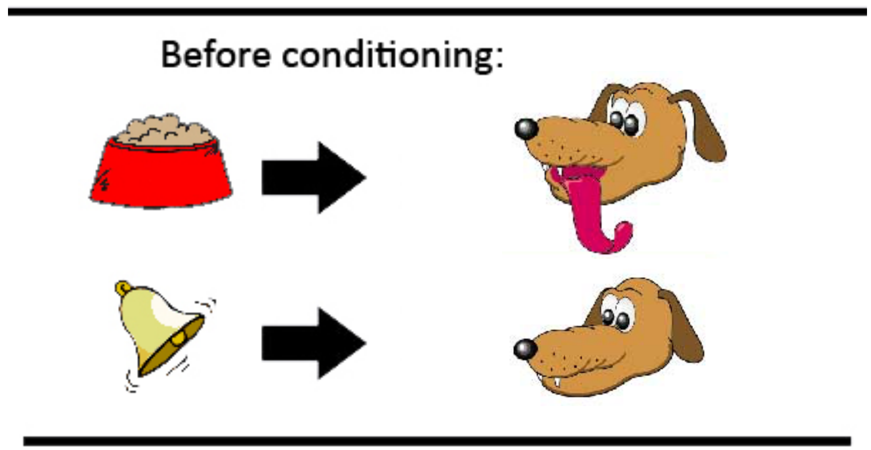
- Unsupervised learning
- Supervised learning
- Reinforcement learning

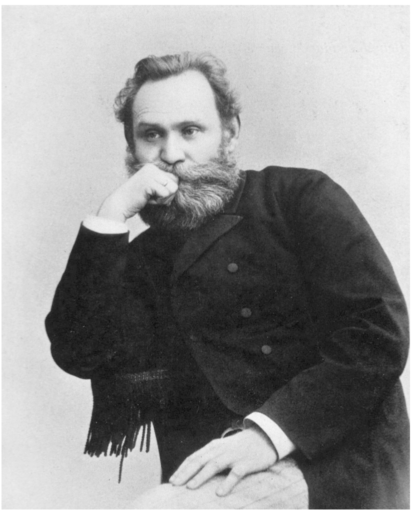
Another active field that combines computation, machine learning, neurophysiology, fMRI

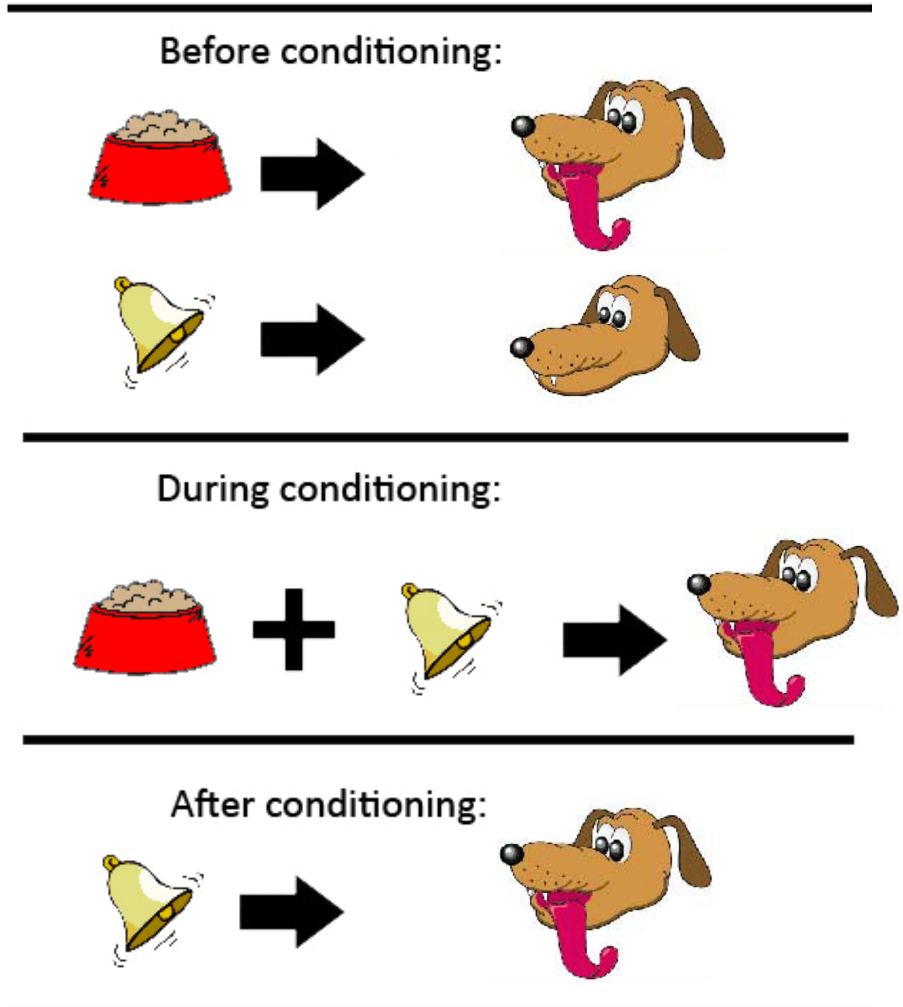
# Pavlov and classical conditioning





# Pavlov and classical conditioning





# Modern terminology

- Stimuli
- Rewards
- Expectations of reward: behavior is learned based on expectations of reward
- Can learn based on consequences of actions (instrumental conditioning); can learn whole sequence of actions (example: maze)

# Rescorla-Wagner rule (1972)

- Can describe classical conditioning and range of related effects
- Based on simple linear prediction of reward associated with a stimulus (error based learning)
- Includes weight updating as in the perceptron rule we did in lab, but we learn from error in predicting reward

# Rescorla-Wagner rule (1972)

- Minimize difference between received reward and predicted reward

- Binary variable u (1 if stimulus is present; 0 if absent)

- Predicted reward v

- Linear weight w

$$v = wu$$

- If stimulus u is present:

$$v = w$$

based on Dayan and Abbott book

# Rescorla-Wagner rule (1972)

- Minimize squared error between received reward r and predicted reward v:

$$(r - v)^2$$

based on Dayan and Abbott book

# Rescorla-Wagner rule (1972)

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$$(r - v)^2$$




In Niv and Schoenbaum 2009

# Rescorla-Wagner rule (1972)

- Minimize squared error between received reward r and predicted reward v:

$$(r - v)^2$$

(average over presentations of stimulus and reward)

- Update weight:

$$w \longrightarrow w + \varepsilon(r - v)u$$

$\varepsilon$ learning rate

Also known as delta learning rule: $\delta = r - v$

- Update weight:

$$w \rightarrow w + \varepsilon(r - v)u$$

- Simpler notation: if a stimulus is presented at trial n (we'll just take u as 1 and set v to w):

$$v_{n+1} = v_n + \epsilon(r_n - v_n)$$

based on Dayan and Abbott book

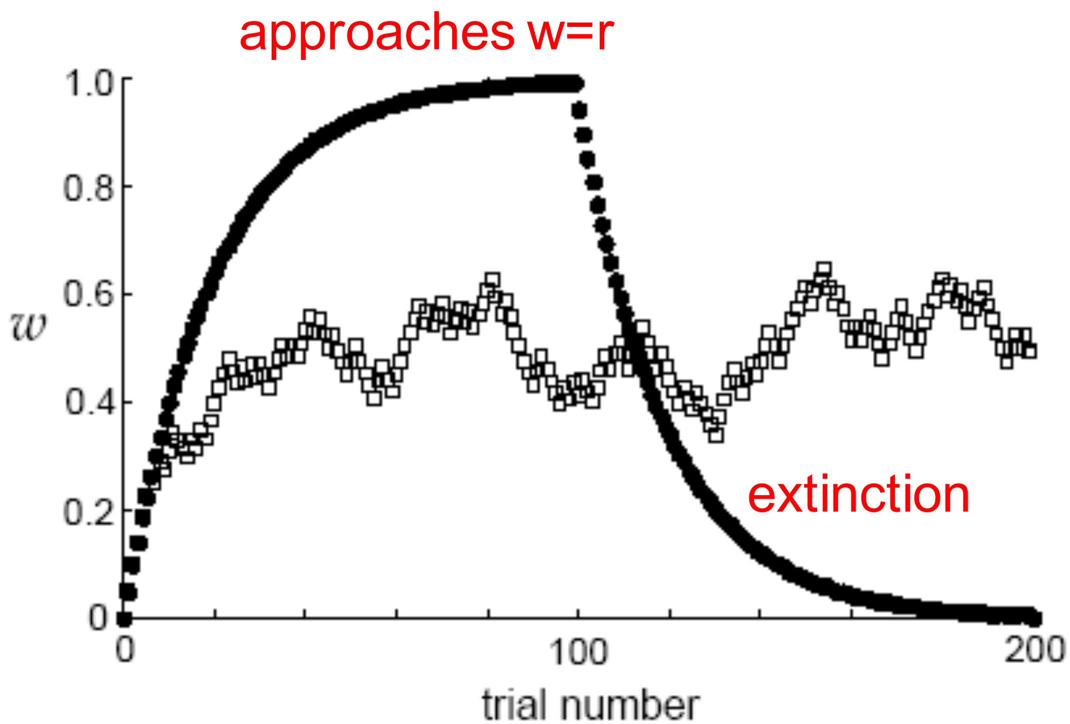
- So if a stimulus is presented at trial n:

$$v_{n+1} = v_n + \epsilon(r_n - v_n)$$

- What happens when learning rate = 1?

- What happens when it is smaller than 1?

# Acquisition and extinction

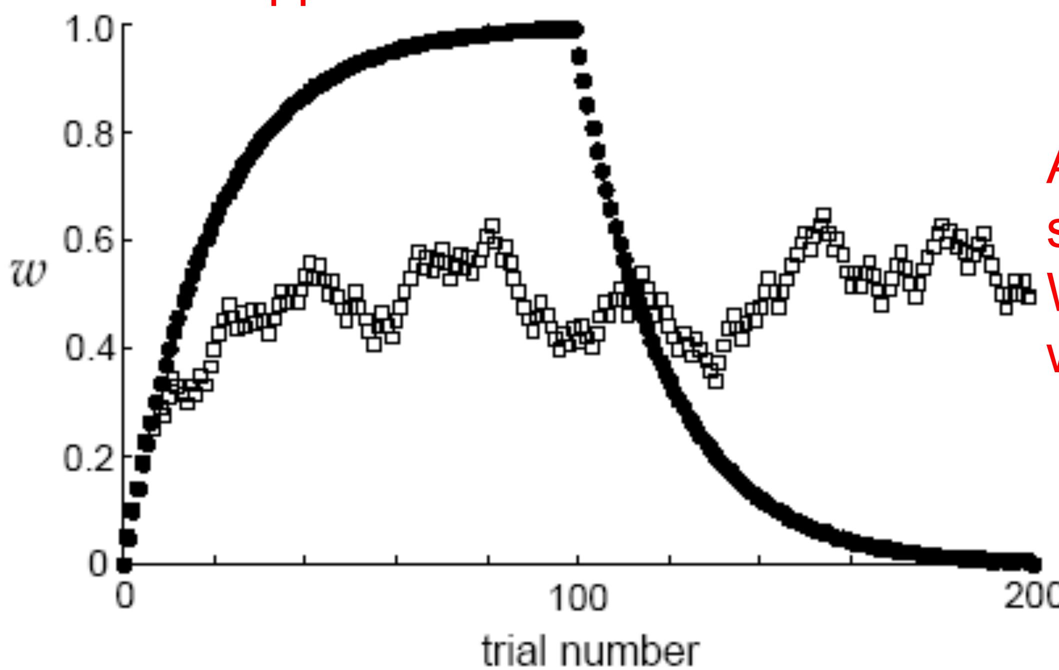



- Solid: First 100 trials: reward (r=1) paired with stimulus; next 100 trials no reward (r=0) paired with stimulus (learning rate .05)
- Dashed: Reward paired with stimulus randomly 50 percent of time

From Dayan and Abbott book

# Acquisition and extinction

approaches w=r

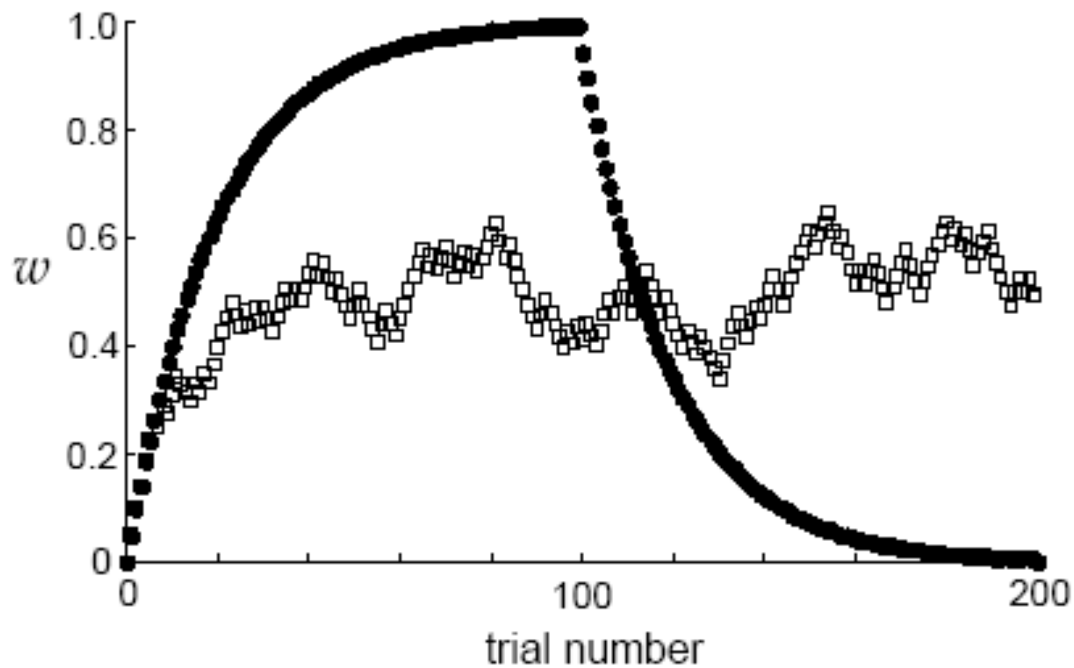



Association of stimulus With reward weaker here

- Solid: First 100 trials: reward (r=1) paired with stimulus; next 100 trials no reward (r=0) paired with stimulus (learning rate .05)
- Dashed: Reward paired with stimulus randomly 50 percent of time

From Dayan and Abbott book

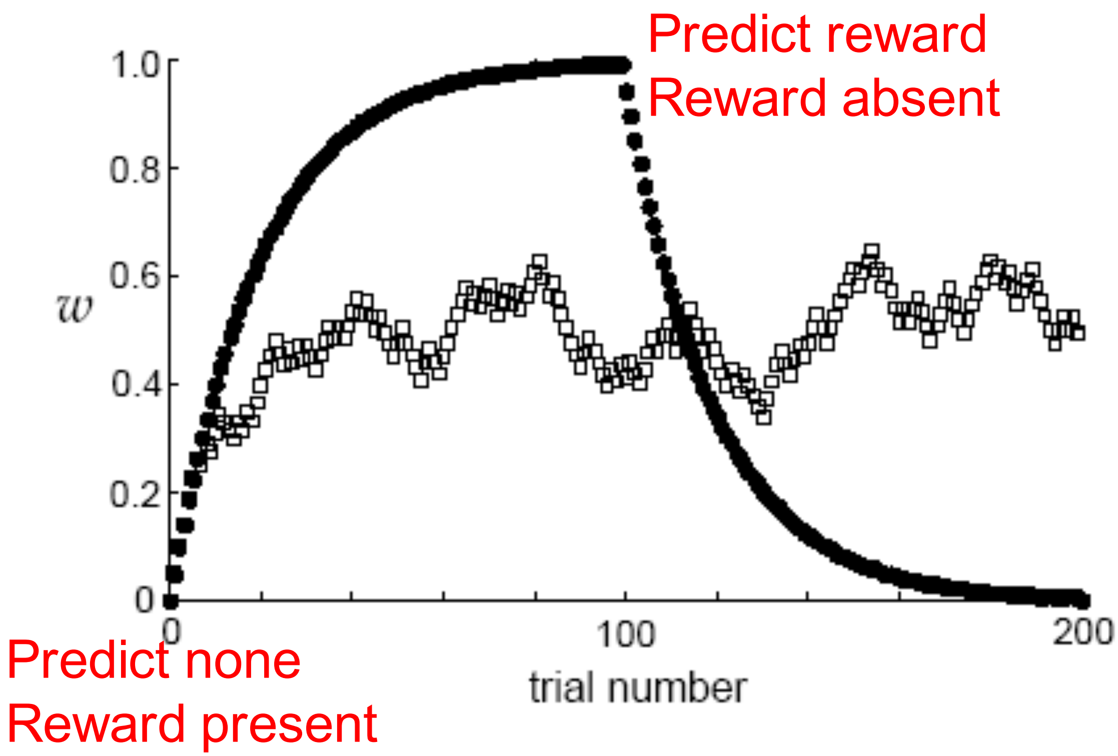
# Acquisition and extinction




- Curves show w over time
- What is the predicted reward v and the error (r-v)?

From Dayan and Abbott book

# Acquisition and extinction

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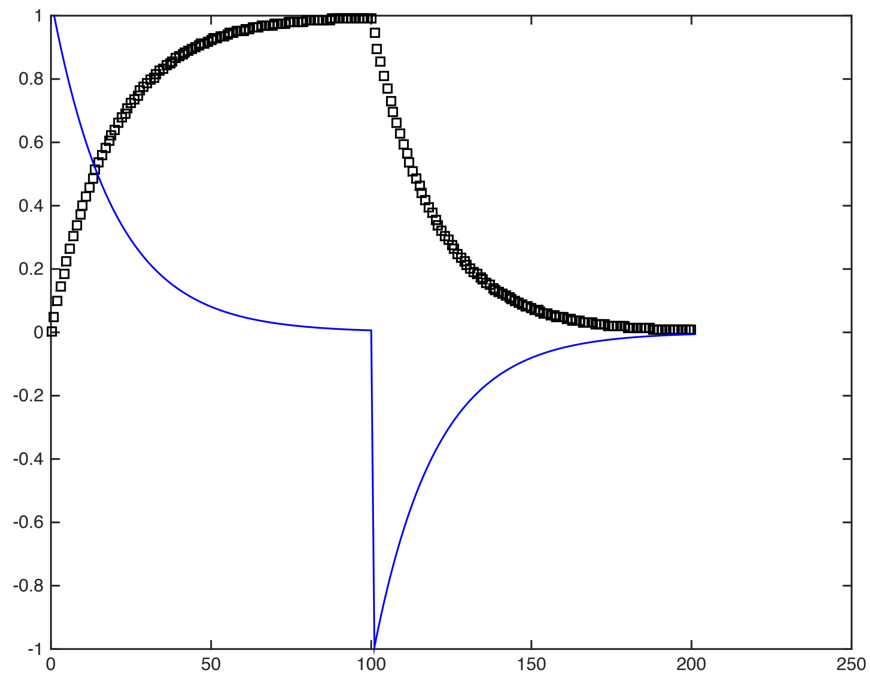
From Dayan and Abbott book

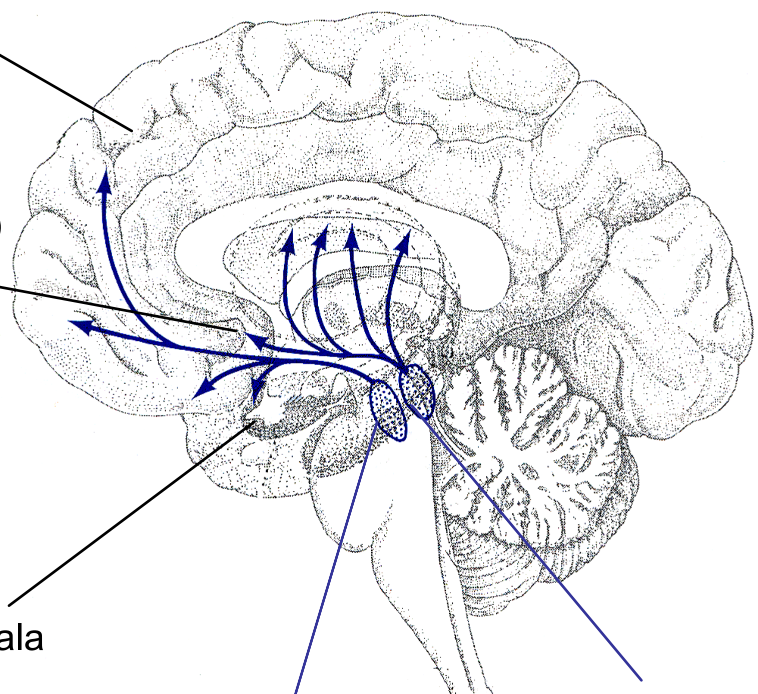
# Acquisition and extinction




- Black curve: v
- Blue curve: (r-v)

From Dayan and Abbott book

# Dopamine areas




From Dayan slides

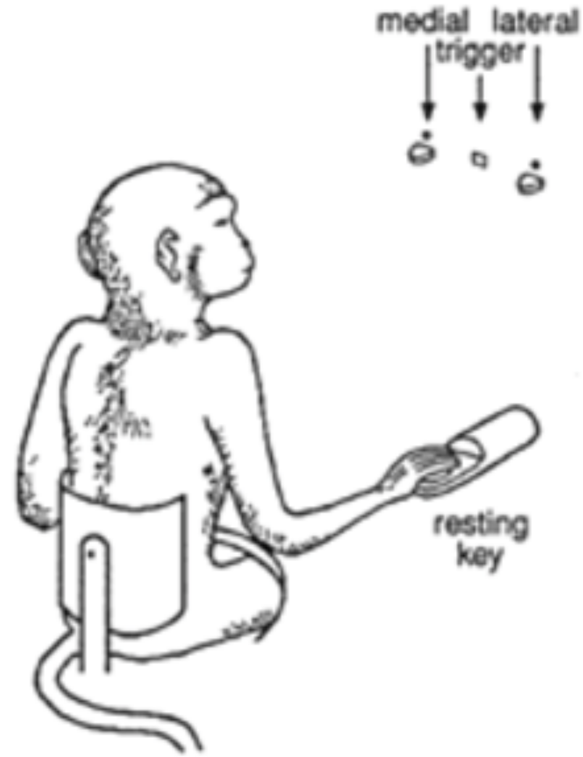
# Dopamine roles?

# Dopamine roles?

Associated with…

- reward (we’ll see prediction error)
- self-stimulation
- motor control (initiation)
- addiction

# VTA Activity of dopaminergic neurons

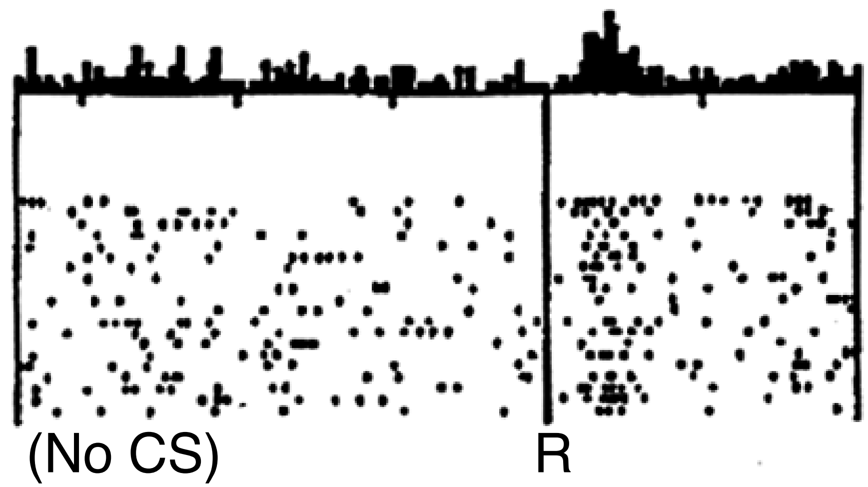



- Monkey trained to respond to light or sound for food and drink rewards (instrumental conditioning)
- Finger on resting key until sound is presented
- Then release key to get reward

Schultz, Dayan, Montague, 1997

# VTA Activity of dopaminergic neurons

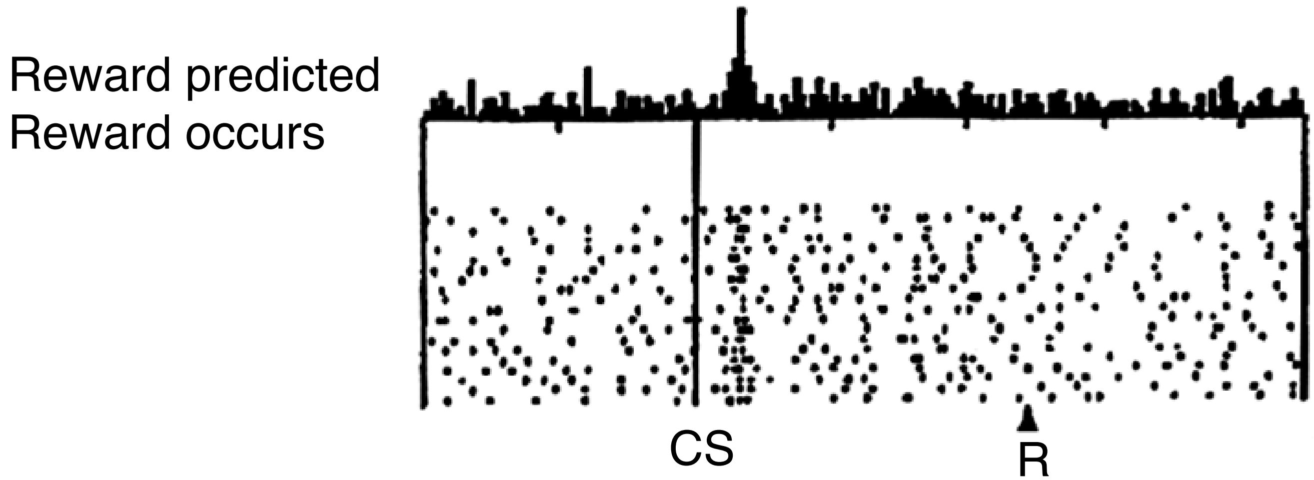
No prediction
Reward occurs




Before learning, reward is given in experiment, but animal does not predict (expect) reward (why is there increased activity after reward?)

Schultz, Dayan, Montague, 1997

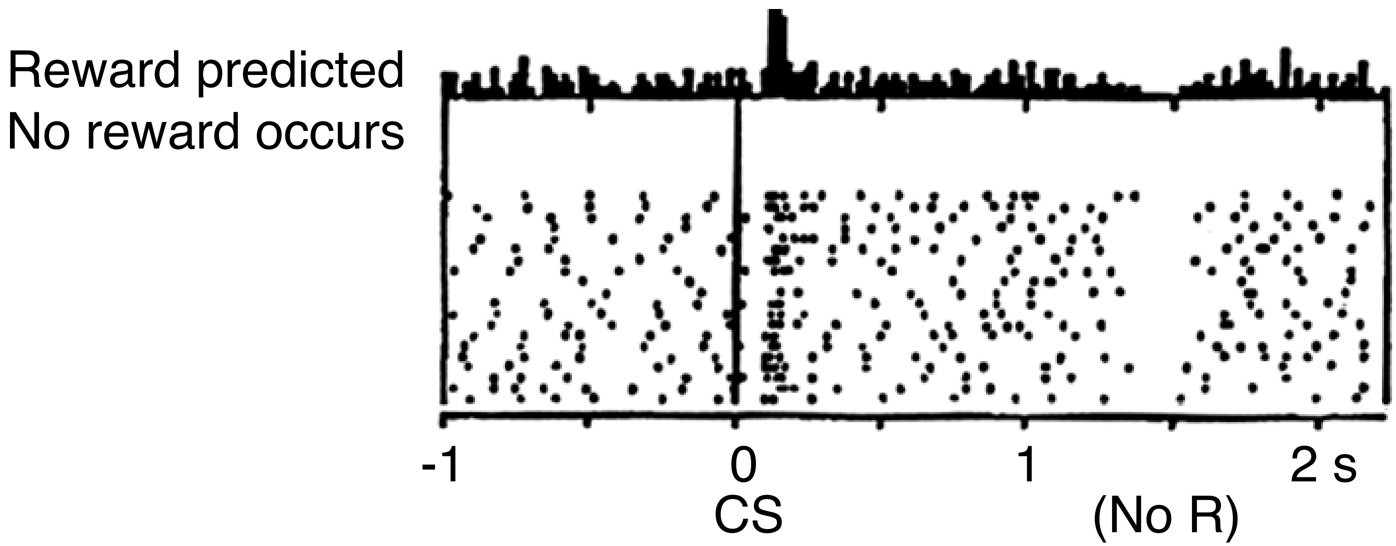
# VTA Activity of dopaminergic neurons




After learning, conditioned stimulus predicts reward, and reward is given in experiment (why is activity fairly uniform after reward?)

Schultz, Dayan, Montague, 1997

# VTA Activity of dopaminergic neurons

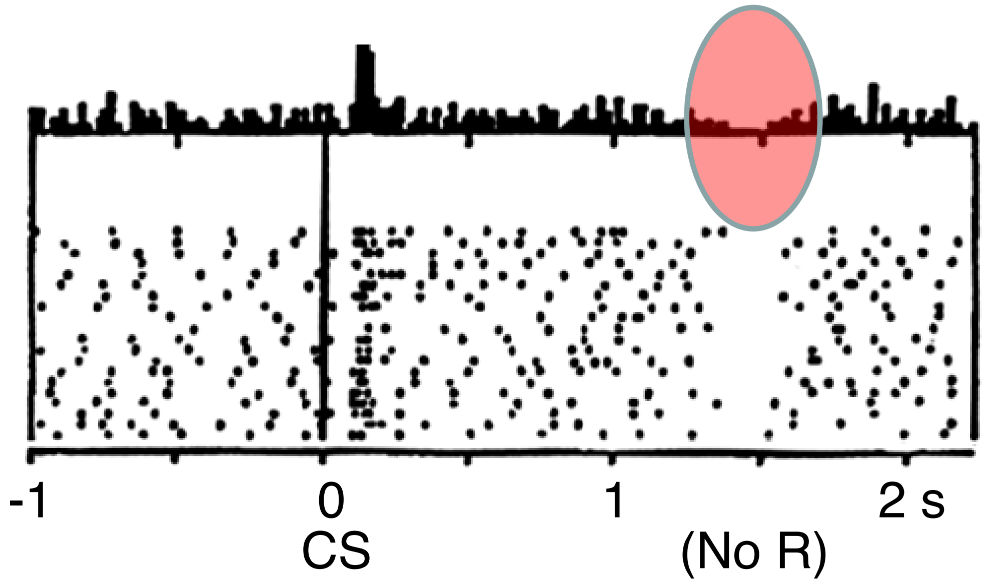



After learning, conditioned stimulus predicts reward so there is an expectation of reward, but no reward is given in the experiment

Schultz, Dayan, Montague, 1997

# VTA Activity of dopaminergic neurons

Reward predicted
No reward occurs

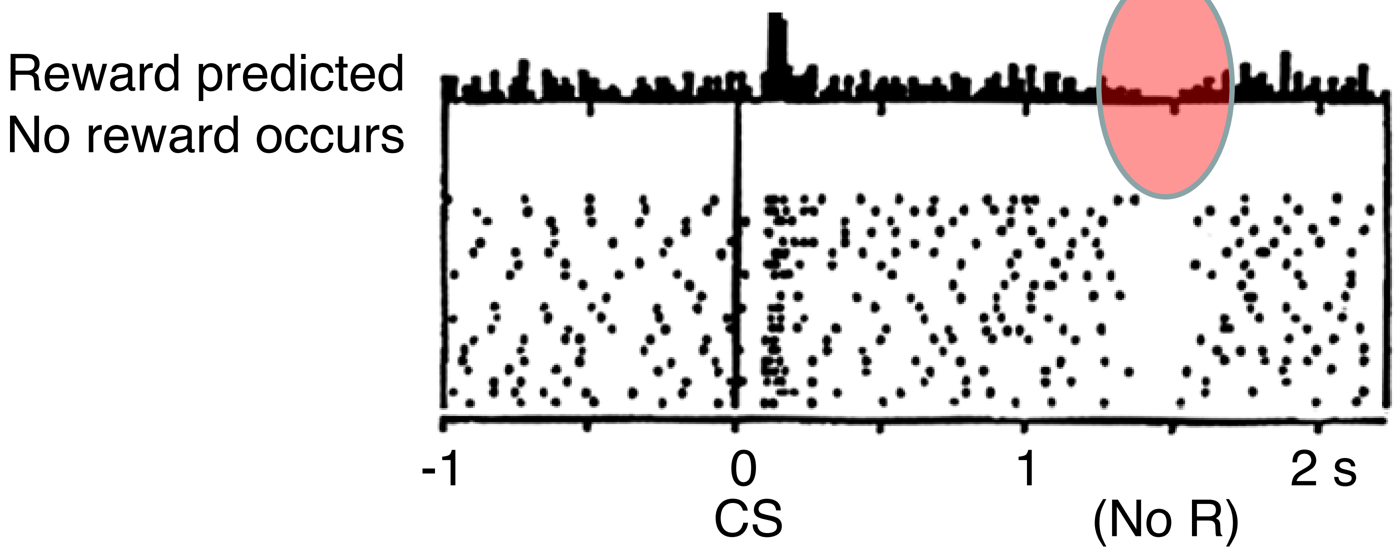



After learning, conditioned stimulus predicts reward so there is an expectation of reward, but no reward is given in the experiment

Why is there a dip? What are these neurons doing?

Schultz, Dayan, Montague, 1997

# VTA Activity of dopaminergic neurons

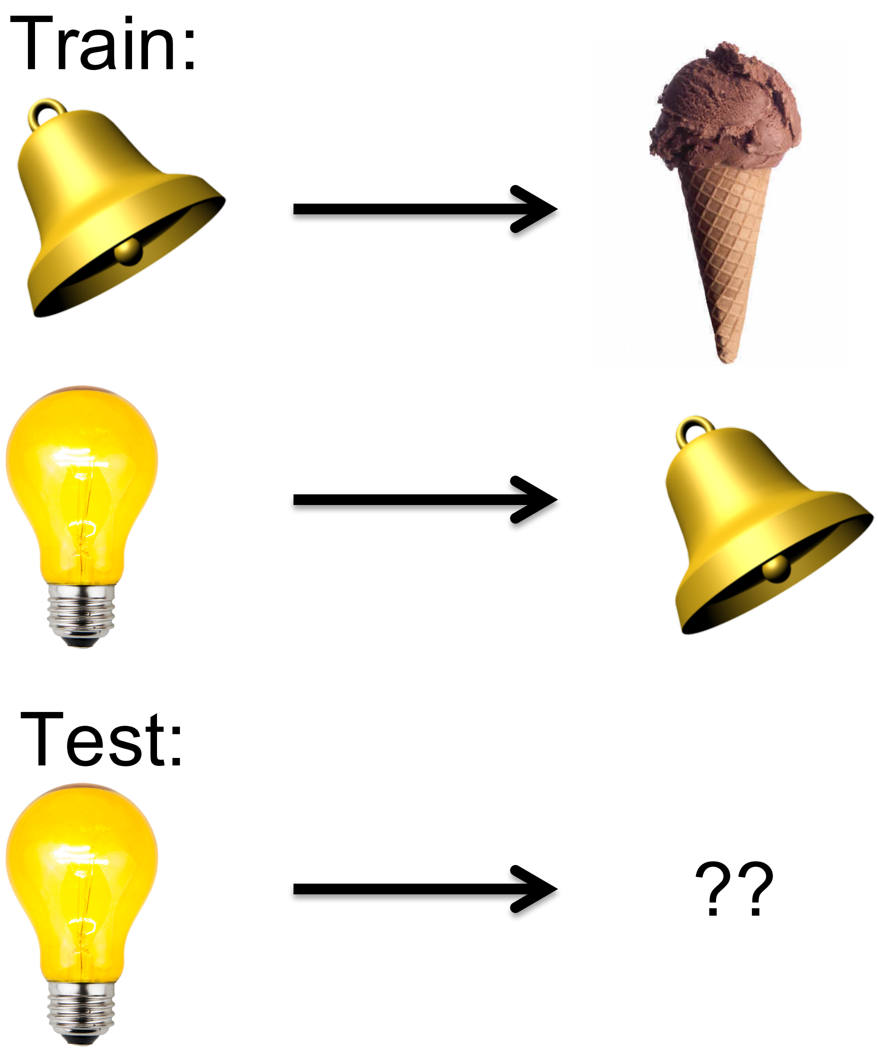



After learning, conditioned stimulus predicts reward so there is an expectation of reward, but no reward is given in the experiment

What are these neurons doing? Prediction error between actual and predicted reward (like r-v)

Schultz, Dayan, Montague, 1997

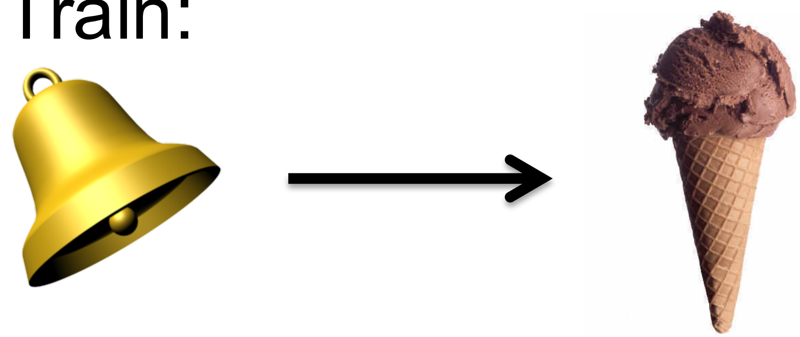
# Shortcomings of Rescorla-Wagner: Example: secondary conditioning

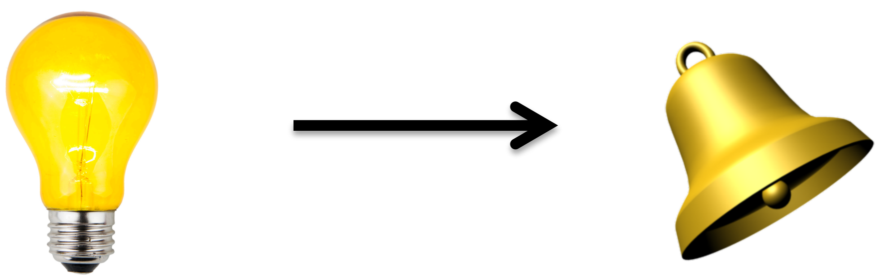



Based on Peter Dayan slides

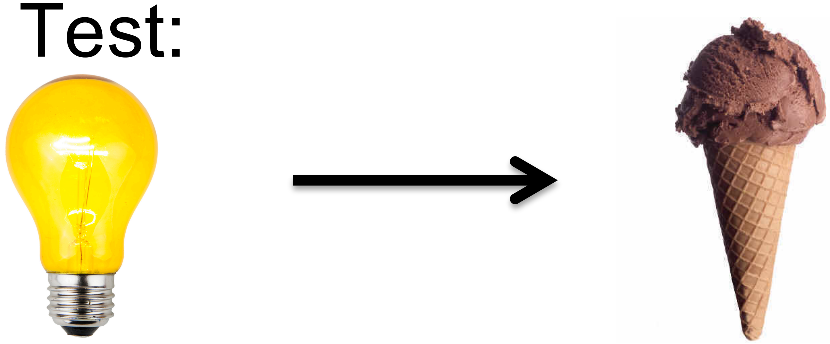
# Shortcomings of Rescorla-Wagner: Example: secondary conditioning

Train:





Test:



Animals learn (more generally, actions that lead to longer term rewards)

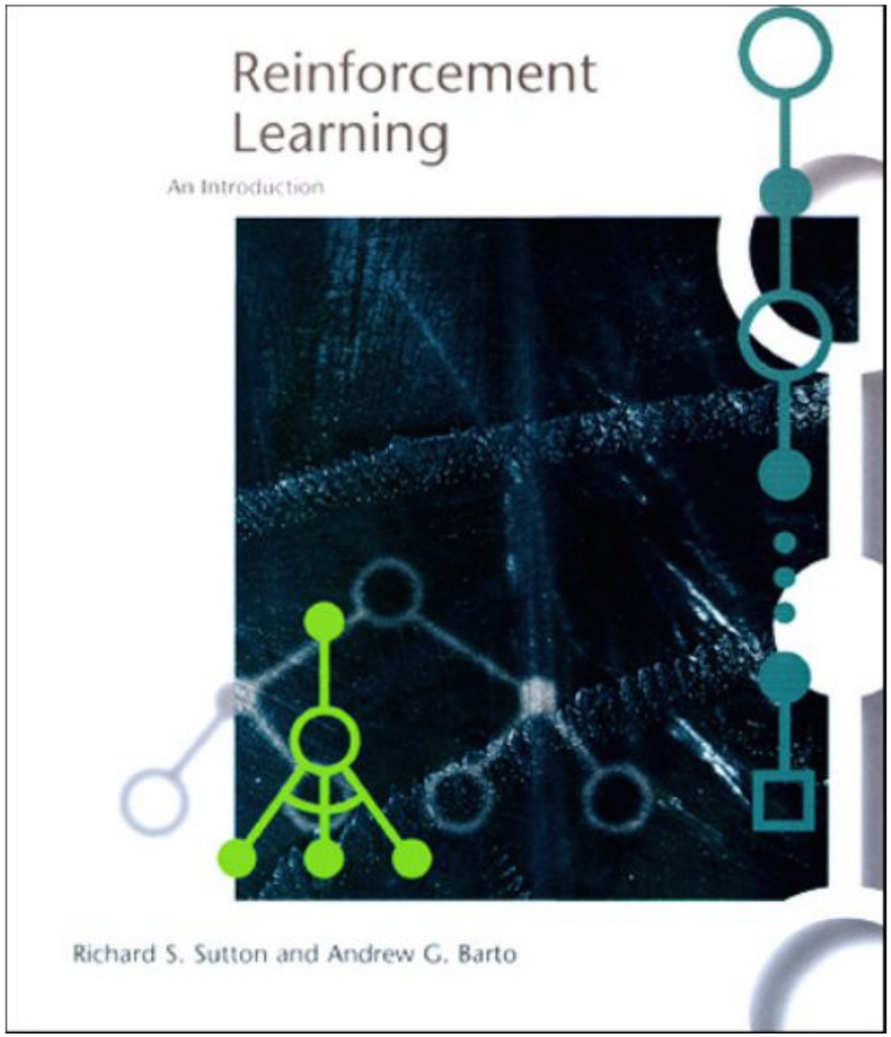
# Shortcomings of Rescorla-Wagner: Example: secondary conditioning

Train:

Test:

Rescorla-Wagner would predict no reward; only predicts immediate reward

# 1990s: Sutton and Barto (Computer Scientists)




Now also
New edition

# 1990s: Sutton and Barto (Computer Scientists)

- Rescorla-Wagner

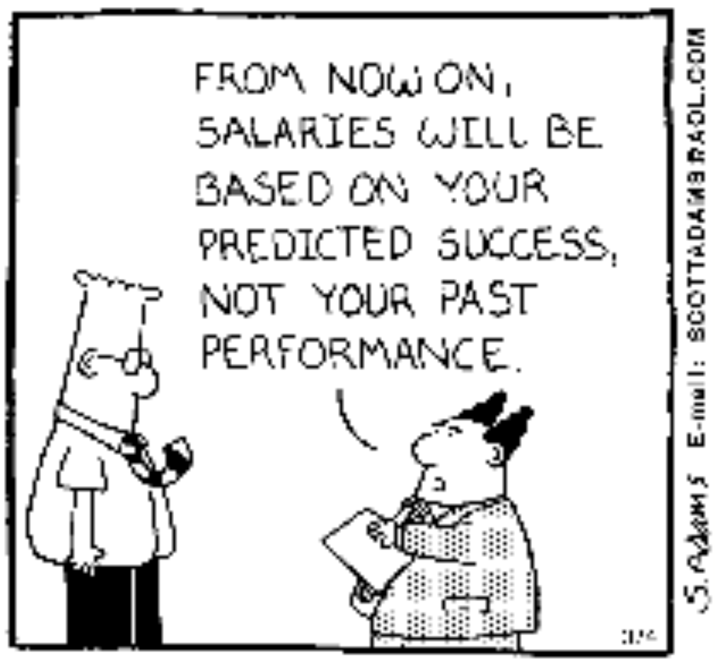
VERSUS

- Temporal Difference Learning:

  Predict value of future rewards (not just current)

# Temporal Difference Learning

- Predict value of future rewards




From Dayan slides

# Temporal Difference Learning

- Predict value of future rewards
- Predictions are useful for behavior
- Generalization of Rescorla-Wagner to real time
- Explains data that Rescorla-Wagner does not

Based on Dayan slides

# Rescorla-Wagner

Want $v_n = r_n$ (here n represents a trial)

Error $\delta_n = r_n - v_n$

$$v_{n+1} = v_n + \varepsilon \delta_n$$

# Temporal Difference Learning

Want $$v_t = r_t + \boxed{r_{t+1} + r_{t+2} + r_{t+3} \ldots}$$

(here t represents time within a trial; reward can come at any time within a trial. Sutton and Barto interpret $v_t$ as the prediction of total future reward expected from time t onward until the end of the trial)

Based on Dayan slides; Daw slides

# Temporal Difference Learning

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Prediction error:

$$\delta_t = (r_t + r_{t+1} + r_{t+2} + r_{t+3} \ldots.) - V_t$$

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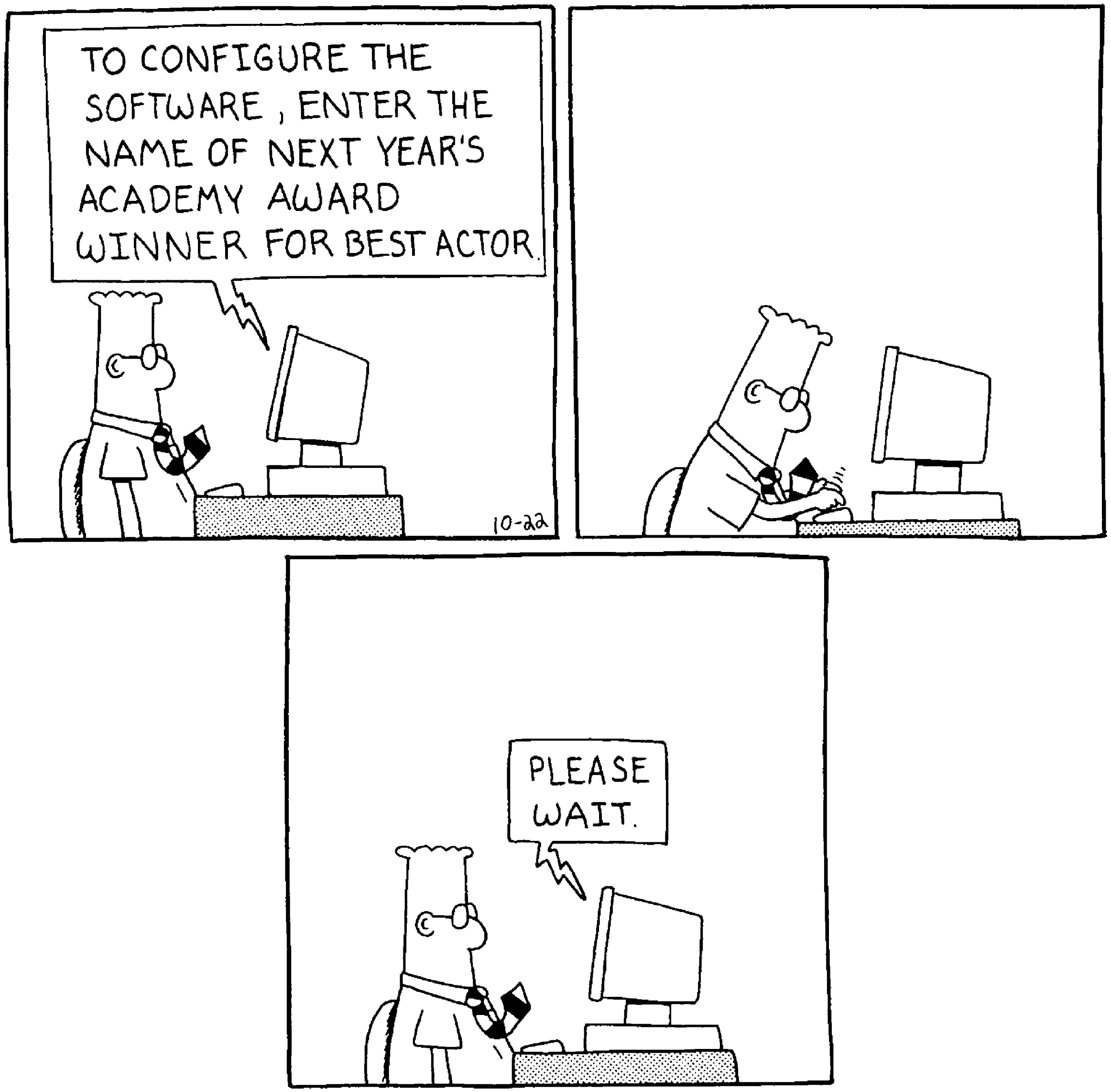
Problem??

Based on Dayan slides; Daw slides






In Niv and Schoenbaum, Trends Cog Sci 2009

# Temporal Difference Learning

Want $$v_t = r_t + r_{t+1} + r_{t+2} + r_{t+3} \ldots.$$

(here t represents time within a trial)

But we don't want to wait forever for all future rewards…

$$r_{t+1}; r_{t+2}; r_{t+3} \ldots.$$

# Temporal Difference Learning

Want $v_t = r_t + r_{t+1} + r_{t+2} + r_{t+3} \ldots$

(here t represents time within a trial)

Recursion "trick":

$$v_t = r_t + v_{t+1}$$

Reward now plus my anticipation now equals total anticipated future

Based on Dayan slides; Daw slides

# Temporal Difference Learning

**From recursion want:**

$$v_t = r_t + v_{t+1}$$

**Error:**

$$\delta_t = r_t + v_{t+1} - v_t$$

Difference between what I anticipate at time t+1 and what I anticipate at time t

# Temporal Difference Learning

From recursion want: $$v_t = r_t + v_{t+1}$$

Error: $$\delta_t = r_t + v_{t+1} - v_t$$

Update: $$v_t \to v_t + \varepsilon(r_t + v_{t+1} - v_t)$$

$$= (1- \ \varepsilon)v_t + \varepsilon(r_t + v_{t+1})$$

# RV versus TD

- Rescorla-Wagner error: (n represents trial)

$$\delta_n = r_n - v_n$$

- Temporal Difference Error: (t is time within a trial)

$$\delta_t = r_t + \boxed{v_{t+1} - v_t}$$

Name comes from!

# Temporal Difference Learning

- Temporal Difference Error: (t is time within a trial)

$$\delta_t = r_t + \boxed{v_{t+1} - v_t}$$

Name comes from!

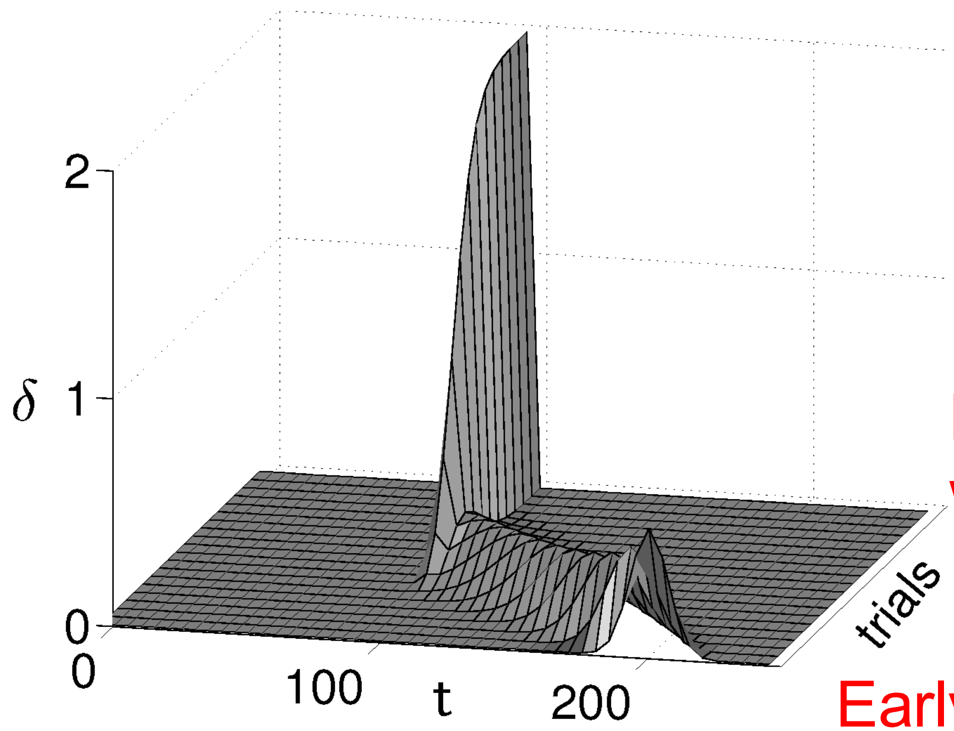
$v_{t+1} = v_t$ Predictions steady

$v_{t+1} > v_t$ Got better

$v_{t+1} < v_t$ Got worse

Based on Daw slides

# Temporal Difference Learning



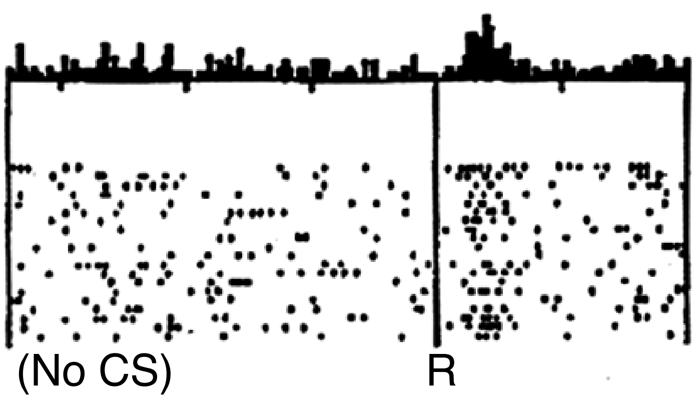

Late trials: error when stimulus

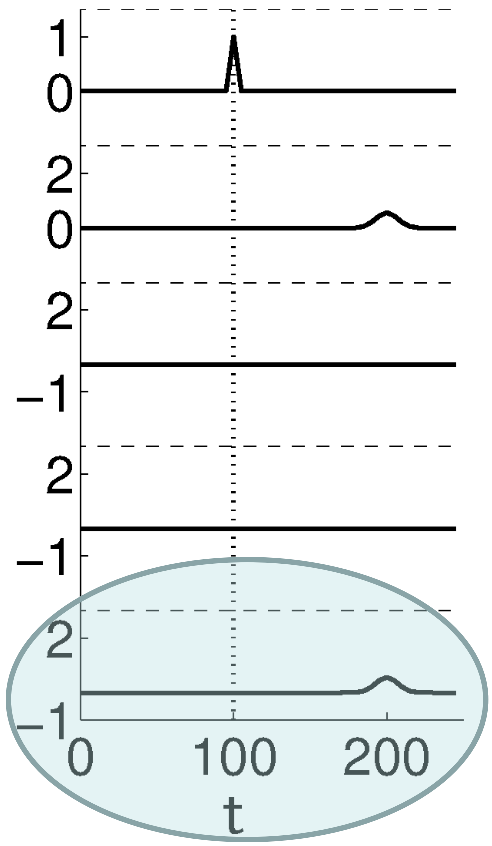
Early trials: error when reward

Dayan and Abbott Book: Surface plot of prediction error (stimulus at 100; reward at 200)

# Temporal Difference Learning

## Before learning




$u_t$ Stimulus

$r_t$ Actual reward

$v_t$ Prediction total reward from t to end of trial

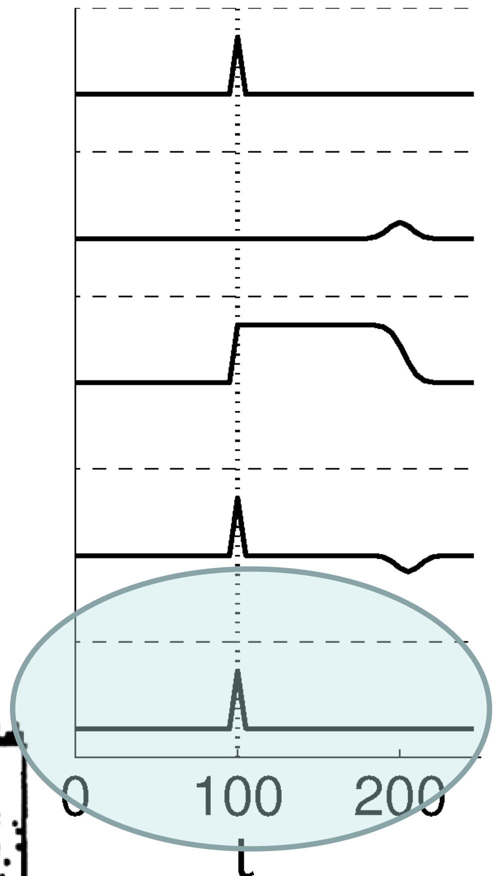
$v_{t+1} - v_t$ Difference t+1 and t

$$\delta_t = r_t + v_{t+1} - v_t$$

Prediction error

# Temporal Difference Learning

## After learning

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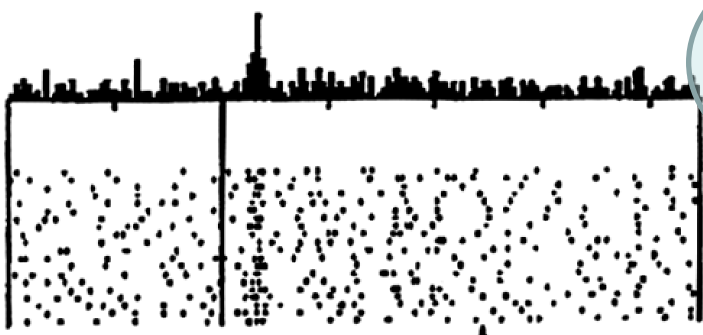
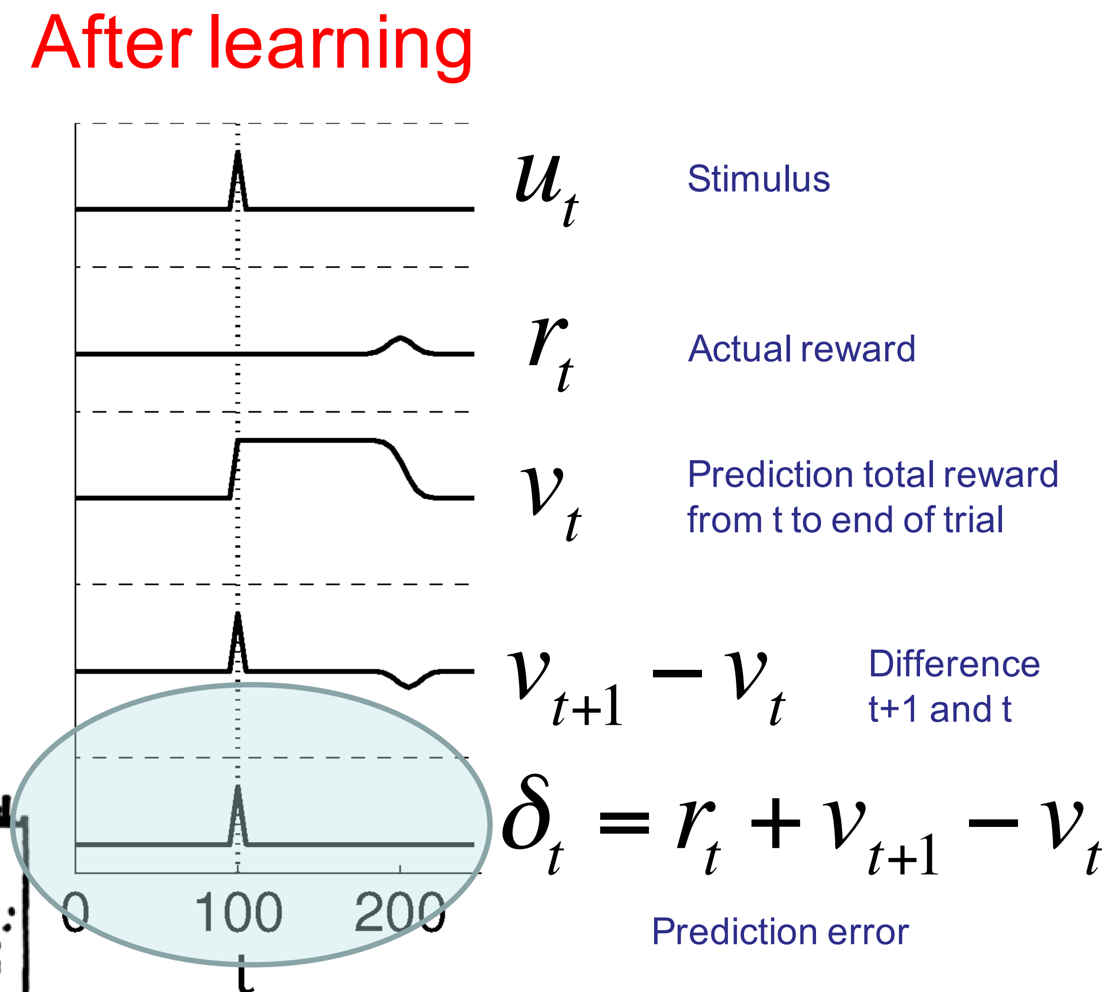
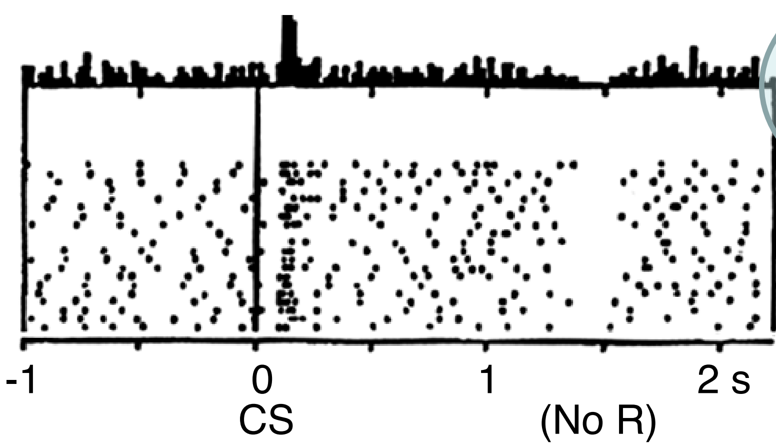
Prediction error



# Temporal Difference Learning

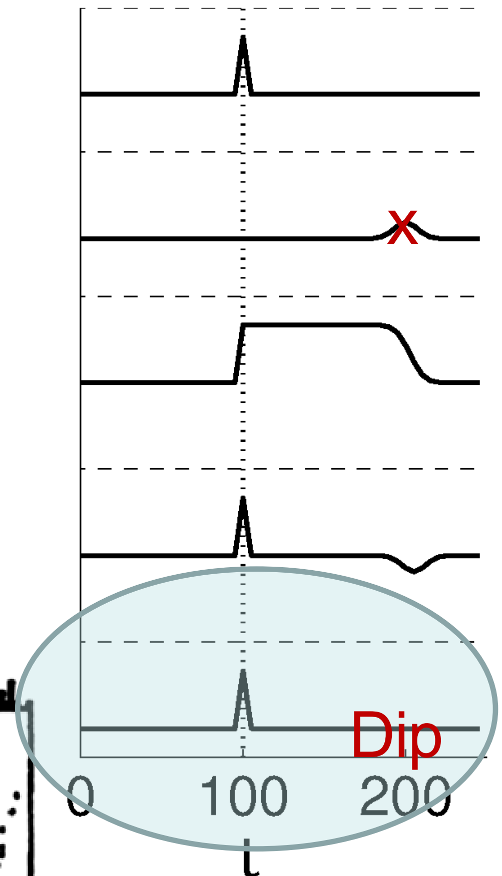



What should change in figure above to match data?

# Temporal Difference Learning

## After learning, and no reward




$u_t$ Stimulus

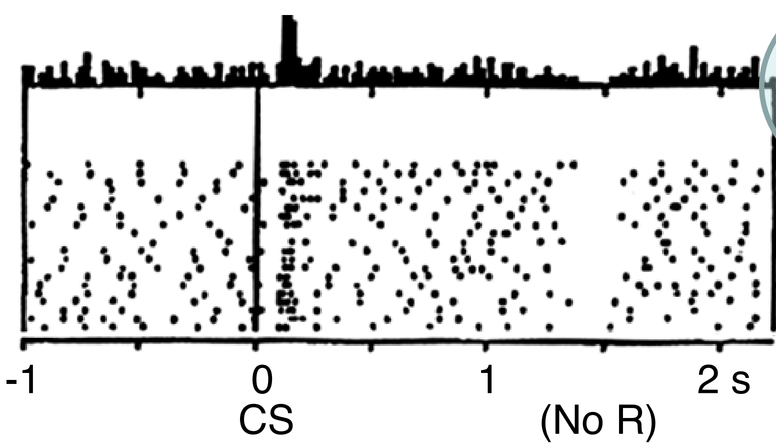
$r_t$ Actual reward

$v_t$ Prediction total reward from t to end of trial

$v_{t+1} - v_t$ Difference t+1 and t

$$\delta_t = r_t + v_{t+1} - v_t$$

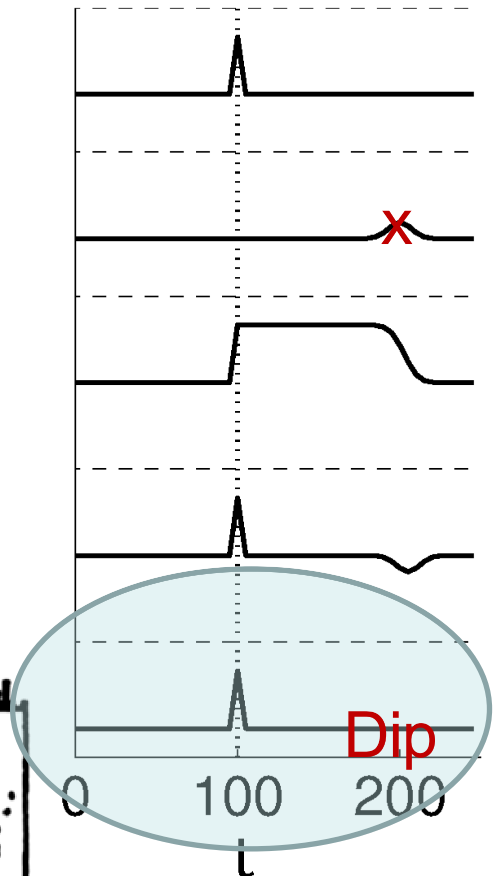
Prediction error




Here reward is 0 and Prediction error should dip

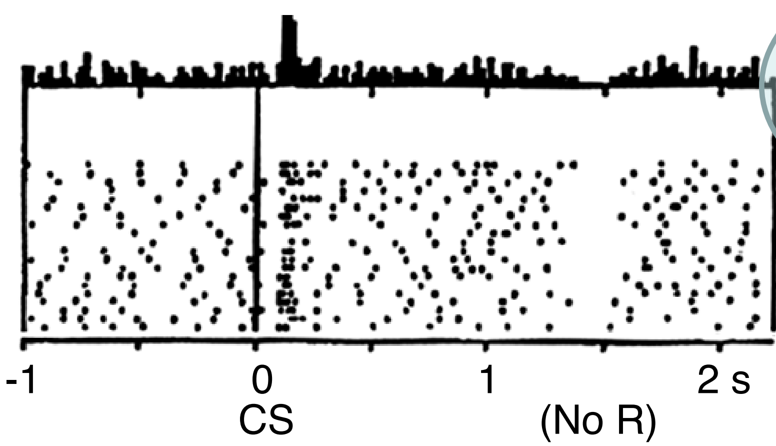
# Temporal Difference Learning

After learning, and no reward




$$\delta_t = r_t + v_{t+1} - v_t$$

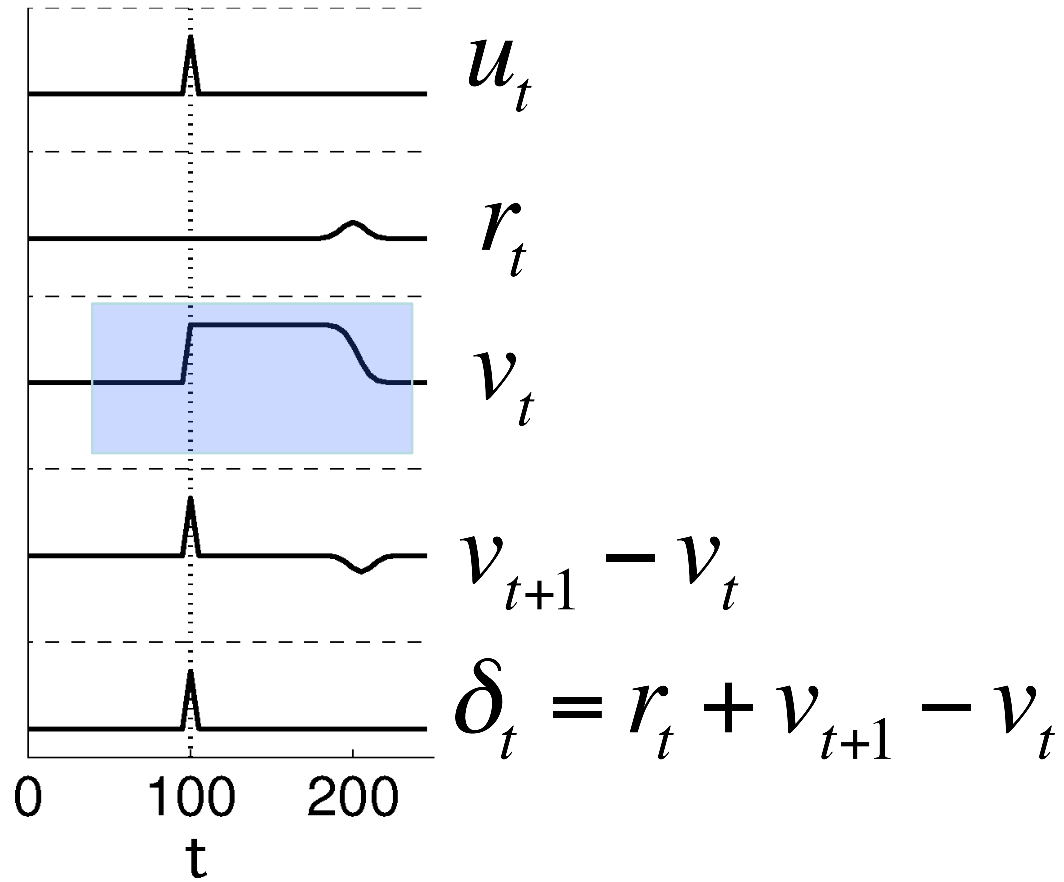
Prediction error




Neural responses related to prediction error

# Temporal Difference Learning

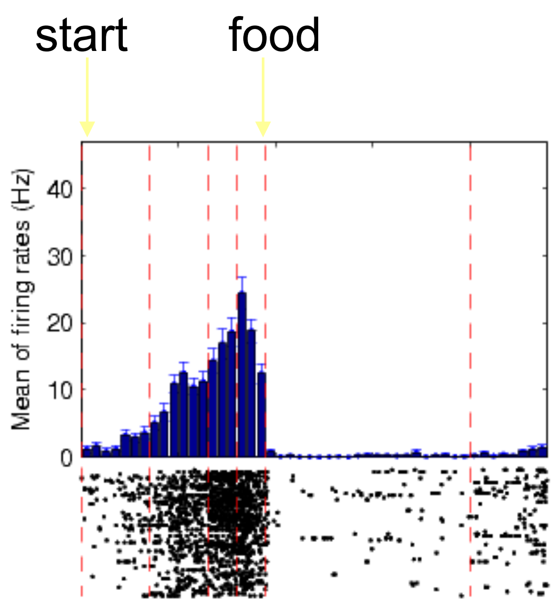
After learning




What about anticipation of future rewards?

# Temporal Difference Learning

Striatal neurons (activity that precedes rewards and changes with learning)




(Daw)

What about anticipation of future rewards?

From Dayan slides

# Summary

Marr's 3 levels:

- Problem: Predict future reward
- Algorithm: Temporal Difference Learning (generalization of Rescorla-Wagner)
- Implementation: Dopamine neurons signaling error in reward prediction

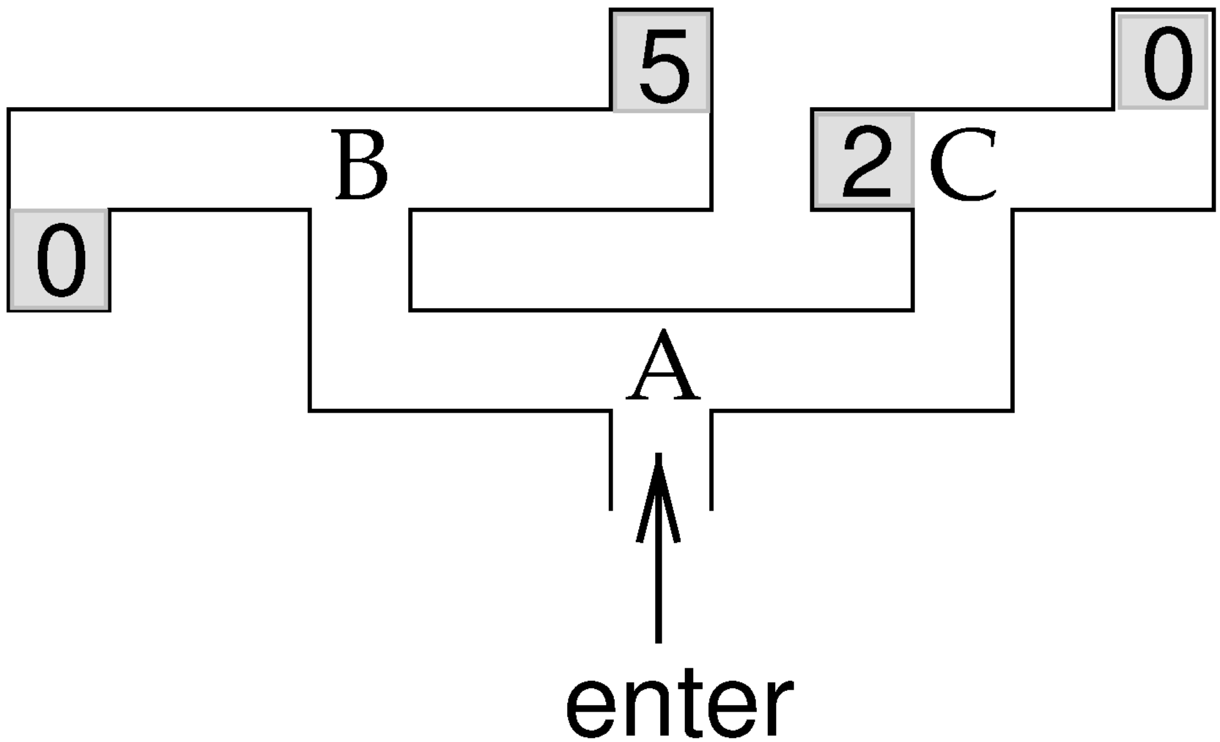
Based on Dayan slides

# What else

- Applied in more sophisticated sequential decision making tasks with future rewards

- Foundation of a lot of active research in Machine Learning, Computational Neuroscience, Biology, Psychology

# More sophisticated tasks




Dayan and Abbott book

Reward based on sequence of actions

# Recent example in machine learning

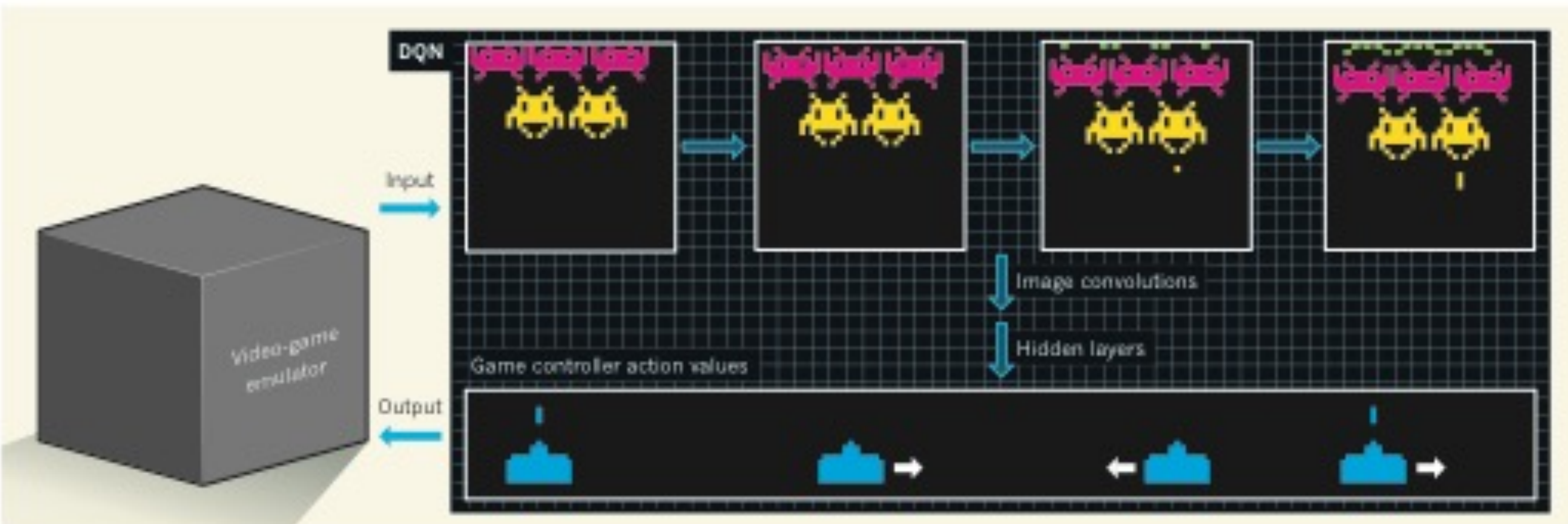
LETTER

doi:10.1038/nature14236

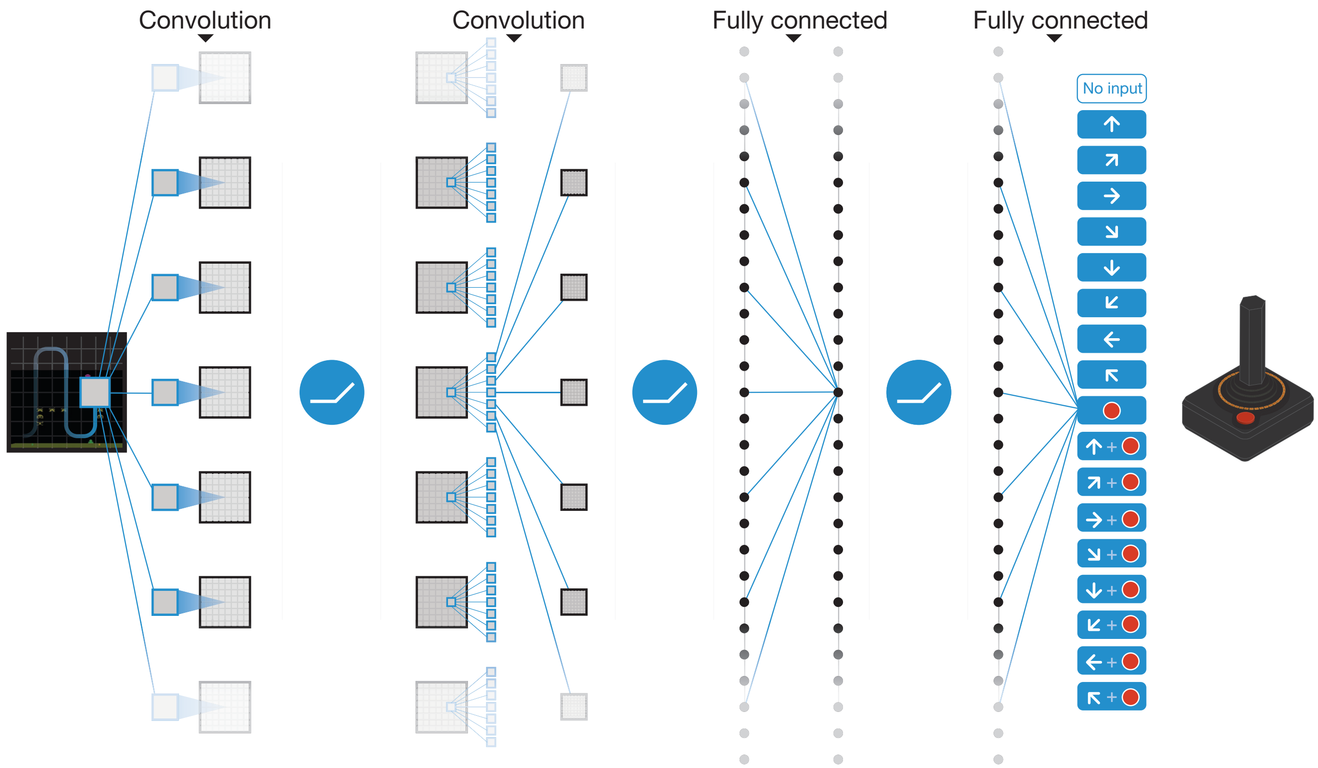
## Human-level control through deep reinforcement learning

Volodymyr Mnih[1]*, Koray Kavukcuoglu[1]*, David Silver[1]*, Andrei A. Rusu[1], Joel Veness[1], Marc G. Bellemare[1], Alex Graves[1], Martin Riedmiller[1], Andreas K. Fidjeland[1], Georg Ostrovski[1], Stig Petersen[1], Charles Beattie[1], Amir Sadik[1], Ioannis Antonoglou[1], Helen King[1], Dharshan Kumaran[1], Daan Wierstra[1], Shane Legg[1] & Demis Hassabis[1]

**Mnih et al**. Nature 518, 529–533; **2015**




**Scholkopf**. News and Views; Nature **2015**

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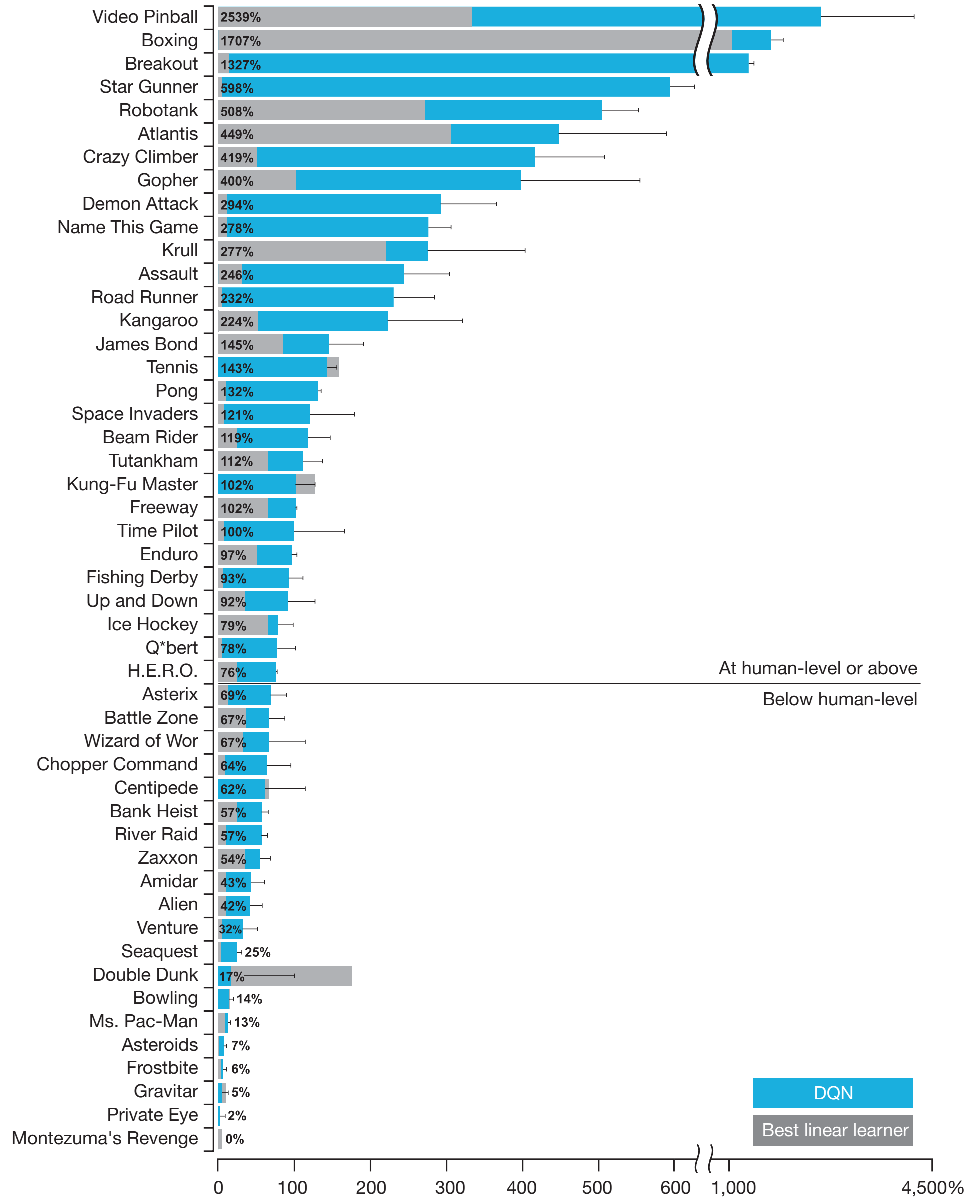


**Mnih et al**. Nature 518, 529–533; **2015**


Silver et al. 2016