Burton Rosenberg
Test 2

April 28, 5:00–6:15

There are five problems each counting equally.

Name: ___________________________________

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On my honor, I have neither given nor received aid on this examination-assignment.

Signature: ________________________________
1. [Network Flows] Negative Cycles

You are given a network $G(V, E)$ with edge costs $w(e)$ for any $e \in E$. These costs can be negative or positive. You would like to find out if there exists a cycle of edges in $G$ such that the weight summed over the edges in the cycle is negative.

Express this as a condition on a network transport problem.
2. [Maximum Matching] Find a maximum matching and minimum cover in the following graph:
3. **[Node Costs and Tree Feasible Solutions]** Give a spanning tree and assign costs, \( \{y_v | v \in V\} \) to the following network. Assume all edge costs are 1.
4. **[APPLICATIONS]** Setup the following problems as a network transport problem.

You are given a network of computers, that is, a graph $G = (V, E)$ where $V$, the set of nodes, represent computers, and $E$, the set of edges, giving communication pathways between computers. Each edge $e \in E$ has a weight, $w(e)$, giving the costs of communication across the pathway. Some of the computers are servers, which we represent as a subset $S$ of $V$. Each non-server, $v \in V \setminus S$ has a demand for disk blocks $b(v)$. A non-server computer $v \in V \setminus S$ will get its blocks over the network from a server by selecting a path to a nearby server, call it $P(v)$, and paying $w(v)b(v)$, where $w(v)$ is the sum of the weights of all edges in the path $P(v)$:

$$w(v) = \sum_{e \in P(v)} w(e).$$

The total disk blocks demanded is:

$$D = \sum_{v \in V \setminus S} b(v).$$

You are to allocate the $D$ blocks to the servers in $S$ such that the total communication costs are lowest, summed over all non-server computers:

$$\sum_{v \in V \setminus S} w(v).$$
5. [Theory] Prove that a connected graph on $n$ vertices is a tree if and only if it has $n - 1$ edges.