Burton Rosenberg
Test 1

February 24, 5:00–6:15

There are four problems each counting equally.

Name: ________________________________

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On my honor, I have neither given nor received aid on this examination-assignment.

Signature: ________________________________
1. [Simplex Method]

Solve the Following LP showing step-by-step the simplex method:

\[
\begin{align*}
\text{max} & \quad x_1 + 2x_2 + x_3 \\
\text{s.t.} & \quad x_1 + x_2 + x_3 \leq 2 \\
& \quad x_1 + x_2 \leq 1 \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]
2. [Duality]

   (a) Give the Dual of the previous LP problem.

   (b) Find the optimal dual solution, using whatever method you wish.

   (c) Demonstrate the Complementary Slackness conditions for your optimal dual/primal solution pair. That is, what should be true and what is true for each of the 5 variable-inequality pairings.
3. [LU Decomposition]

(a) Use Gaussian Elimination with partial pivoting to decompose,

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
2 & 0 & 2 \\
0 & 3 & 3
\end{bmatrix},
\]

into the product,

\[
L_3 P_3 L_2 P_2 L_1 P_1 A = U,
\]

where \( L_i \) are column \( i \) eta-matrices, \( P_i \) are permutation matrices, and \( U \) is upper triangular with 1’s down the diagonal.

(b) Use back substitution and your decomposition to find \( x_1, x_2, x_3 \) real numbers which satisfy,

\[
\begin{align*}
x_1 + x_2 + x_3 &= 1 \\
x_1 + x_3 &= 1/2 \\
x_2 + x_3 &= 1/3
\end{align*}
\]
4. [Theory]

Prove that the product $AB$ of two square matrices is nonsingular if and only if both $A$ and $B$ are nonsingular.