A Tableau Decision Procedure for Propositional Intuitionistic Logic

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Outline

1. **Preliminaries**
   - Tableau calculus
   - Branching and Backtracking
   - Formulas groups
2. **Optimizations**
   - Bounding depth: opt1
   - Bounding branching: opt2
   - Avoiding backtracking: opt3
3. **PITP**
   - About the implementation
   - ILTP Library
4. **Conclution and Future works**

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Calculus

Tableau calculus

- Enhancement of Fitting tableaux
- Related to tableau/sequent calculi: Dyckhoff, Hudelmaier, Miglioli, Moscato and Ornaghi
- Key words: Duplication free/contraction free, PSPACE-completeness

Tableau vs Kripke semantics

A tableau proof for a formula is the attempt to build a model satisfying the formula.

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Tableau vs Kripke semantics

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Rules

$$\frac{S, \mathbf{T}(A \wedge B)}{S, \mathbf{T}A, \mathbf{T}B}\mathbf{T}\wedge \qquad \frac{S, \mathbf{F}(A \wedge B)}{S, \mathbf{F}A | S, \mathbf{F}B}\mathbf{F}\wedge \qquad \frac{S, \mathbf{F_c}(A \wedge B)}{S_c, \mathbf{F_c}A | S_c, \mathbf{F_c}B}\mathbf{F_c}\wedge$$

$$\frac{S, \mathbf{T}(A \vee B)}{S, \mathbf{T}A | S, \mathbf{T}B}\mathbf{T}\vee \qquad \frac{S, \mathbf{F}(A \vee B)}{S, \mathbf{F}A, \mathbf{F}B}\mathbf{F}\vee \qquad \frac{S, \mathbf{F_c}(A \vee B)}{S, \mathbf{F_c}A, \mathbf{F_c}B}\mathbf{F_c}\vee$$

$$\frac{S, \mathbf{T}A, \mathbf{T}(A \rightarrow B)}{S, \mathbf{T}A, \mathbf{T}B}\mathbf{T} \rightarrow Atom, \text{ with } A \text{ an atom}$$

$$\frac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{T}A, \mathbf{F}B}\mathbf{F} \rightarrow \qquad \frac{S, \mathbf{F_c}(A \rightarrow B)}{S_c, \mathbf{T}A, \mathbf{F_c}B}\mathbf{F_c} \rightarrow$$

$$\frac{S, \mathbf{T}(\neg A)}{S, \mathbf{F_c}A}\mathbf{T}\neg \qquad \frac{S, \mathbf{F}(\neg A)}{S_c, \mathbf{T}A}\mathbf{F}\neg \qquad \frac{S, \mathbf{F_c}(\neg A)}{S_c, \mathbf{T}A}\mathbf{F_c}\neg$$

$$\frac{S, \mathbf{T}((A \wedge B) \rightarrow C)}{S, \mathbf{T}(A \rightarrow (B \rightarrow C))}\mathbf{T} \rightarrow \wedge \qquad \frac{S, \mathbf{T}(\neg A \rightarrow B)}{S_c, \mathbf{T}A | S, \mathbf{T}B}\mathbf{T} \rightarrow \neg$$

$$\frac{S, \mathbf{T}((A \vee B) \rightarrow C)}{S, \mathbf{T}(A \rightarrow p), \mathbf{T}(B \rightarrow p), \mathbf{T}(p \rightarrow C)}\mathbf{T} \rightarrow \vee$$

$$\frac{S, \mathbf{T}((A \rightarrow B) \rightarrow C)}{S_c, \mathbf{T}A, \mathbf{F}p, \mathbf{T}(p \rightarrow C), \mathbf{T}(B \rightarrow p) | S, \mathbf{T}C}\mathbf{T} \rightarrow\rightarrow$$

where $S_c = \{\mathbf{T}A | \mathbf{T}A \in S\} \cup \{\mathbf{F_c}A | \mathbf{F_c}A \in S\}$ and p is a new atom

Branching

The rules having more than one conclusion give rise to branches ($\dfrac{S, \mathbf{T}(A \vee B)}{S, \mathbf{T}A | S, \mathbf{T}B} \mathbf{T}\vee$). Thus the search space consists of a proof whose branches have to be visited by the decision procedure.

Backtracking

In intuitionistic logic the order in which the rules are applied is relevant and affect the completeness. If the choice of a swff does not give a closed proof table, one has to backtrack and try with another swff (e.g. $\dfrac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{T}A, \mathbf{F}B} \mathbf{F} \rightarrow$).

Fact

The PSPACE-completeness of intuitionistic logic (Statman:79) suggests that backtracking cannot be eliminated.

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Groups

Six group

The formulas are divided in six groups according to their behavior with respect to branching and backtracking

- $\mathcal{C}_1 = \{\mathbf{T}(A \wedge B), \mathbf{F}(A \vee B), \mathbf{F_c}(A \vee B), \mathbf{T}(\neg A),$ $\mathbf{T}(p \rightarrow A)$ with p an atom, $\mathbf{T}((A \wedge B) \rightarrow C),$ $\mathbf{T}((A \vee B) \rightarrow C)\};$
- $\mathcal{C}_2 = \{\mathbf{T}(A \vee B), \mathbf{F}(A \wedge B);$
- $\mathcal{C}_3 = \{\mathbf{F}(\neg A), \mathbf{F}(A \rightarrow B)\};$
- $\mathcal{C}_4 = \{\mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(\neg A \rightarrow B)\};$
- $\mathcal{C}_5 = \{\mathbf{F_c}(A \rightarrow B), \mathbf{F_c}(\neg A)\};$
- $\mathcal{C}_6 = \{\mathbf{F_c}(A \wedge B)\}.$

Groups

$$\frac{S, \mathbf{T}(A \wedge B)}{S, \mathbf{T}A, \mathbf{T}B}\mathbf{T}\wedge \qquad \frac{S, \mathbf{F}(A \wedge B)}{S, \mathbf{F}A | S, \mathbf{F}B}\mathbf{F}\wedge \qquad \frac{S, \mathbf{F_c}(A \wedge B)}{S_c, \mathbf{F_c}A | S_c, \mathbf{F_c}B}\mathbf{F_c}\wedge$$

$$\frac{S, \mathbf{T}(A \vee B)}{S, \mathbf{T}A | S, \mathbf{T}B}\mathbf{T}\vee \qquad \frac{S, \mathbf{F}(A \vee B)}{S, \mathbf{F}A, \mathbf{F}B}\mathbf{F}\vee \qquad \frac{S, \mathbf{F_c}(A \vee B)}{S, \mathbf{F_c}A, \mathbf{F_c}B}\mathbf{F_c}\vee$$

$$\frac{S, \mathbf{T}A, \mathbf{T}(A \rightarrow B)}{S, \mathbf{T}A, \mathbf{T}B}\mathbf{T} \rightarrow Atom, \text{ with } A \text{ an atom}$$

$$\frac{S, \mathbf{F}(A \rightarrow B)}{S_c, \mathbf{T}A, \mathbf{F}B}\mathbf{F} \rightarrow \qquad \frac{S, \mathbf{F_c}(A \rightarrow B)}{S_c, \mathbf{T}A, \mathbf{F_c}B}\mathbf{F_c} \rightarrow$$

$$\frac{S, \mathbf{T}(\neg A)}{S, \mathbf{F_c}A}\mathbf{T}\neg \qquad \frac{S, \mathbf{F}(\neg A)}{S_c, \mathbf{T}A}\mathbf{F}\neg \qquad \frac{S, \mathbf{F_c}(\neg A)}{S_c, \mathbf{T}A}\mathbf{F_c}\neg$$

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where $S_c = \{\mathbf{T}A | \mathbf{T}A \in S\} \cup \{\mathbf{F_c}A | \mathbf{F_c}A \in S\}$ and p is a new atom

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$$\frac{S, \mathbf{T}(A \lor B)}{S, \mathbf{T}A | S, \mathbf{T}B}\mathbf{T}\lor \qquad \frac{S, \mathbf{F}(A \lor B)}{S, \mathbf{F}A, \mathbf{F}B}\mathbf{F}\lor \qquad \frac{S, \mathbf{F_c}(A \lor B)}{S, \mathbf{F_c}A, \mathbf{F_c}B}\mathbf{F_c}\lor$$

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Outline

What happen if the CL-model underling S realizes S?

Example

$S = \{\mathbf{T}\,A, \mathbf{F}\,B, \mathbf{T}((A \rightarrow B) \rightarrow C)\}$

$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$

$\sigma \rhd S$

$\sigma \models A$	$\sigma \rhd \mathbf{T}A$
$\sigma \not\models B$	$\sigma \rhd \mathbf{F}B$
$\sigma \models (A \rightarrow B) \rightarrow C$	$\sigma \rhd \mathbf{T}((A \rightarrow B) \rightarrow C)$

The Kripke model coinciding with the classical model σ realizes S.

we do not need to go on with the proof.

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and what happen if CL-model underling S does not realize S

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$\sigma \not\rhd S$

$\sigma \not\models A \rightarrow B \;\Big|\; \sigma \not\rhd \mathbf{T}(A \rightarrow B)$

we cannot conclude that S is closed

BUT...

and what happen if CL-model underling S does not realize S

Example

$S = \{\mathbf{T}\,A, \mathbf{F}\,B, \mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(A \rightarrow B)\}$
$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$
$\sigma \not\triangleright S$
$\sigma \not\models A \rightarrow B \mid \sigma \not\triangleright \mathbf{T}(A \rightarrow B)$

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$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$

$\sigma \not\triangleright S$

$\sigma \not\models A \rightarrow B \mid \sigma \not\triangleright \mathbf{T}(A \rightarrow B)$

we cannot conclude that S is closed

BUT...

if every $\mathcal{PV}$ in S is signed T or $\mathbf{F_c}$

Example

$S = \{\mathbf{T}\,A, \mathbf{F_c}\,B, \mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(A \rightarrow B), \mathbf{F_c}C\}$

$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$

$\sigma \not\triangleright S$

$\sigma \not\models A \rightarrow B \;\Big|\; \sigma \not\triangleright \mathbf{T}(A \rightarrow B)$

There is no Kripke model realizing S

we do not need to proceed with the proof.

if every $\mathcal{PV}$ in S is signed T or $\mathbf{F_c}$

Example

$S = \{\mathbf{T}\,A, \mathbf{F_c}\,B, \mathbf{T}((A \rightarrow B) \rightarrow C), \mathbf{T}(A \rightarrow B), \mathbf{F_c}C\}$

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if every $\mathcal{PV}$ in S is signed T or $\mathbf{F_c}$

Example

$S = \{\mathbf{T}\,A, \mathbf{F_c}\,B, \mathbf{T}((A \to B) \to C), \mathbf{T}(A \to B), \mathbf{F_c}C\}$

$\sigma = \{A \to \mathit{true}, B \to \mathit{false}, C \to \mathit{false}\}$

$\sigma \not\rhd S$

$\sigma \not\models A \to B \mid \sigma \not\rhd \mathbf{T}(A \to B)$

There is no Kripke model realizing S

we do not need to proceed with the proof.

Bounding branching: opt2

swffs whose intuitionistic truth coincides with classical truth

Example

$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}(B \vee C), \mathbf{T}A\}$

$\sigma = \{A \rightarrow \mathit{true}, B \rightarrow \mathit{false}, C \rightarrow \mathit{false}\}$

$\sigma \not\models A \wedge B$	$\sigma \triangleright \mathbf{F}(A \wedge B)$
$\sigma \not\models A \wedge C$	$\sigma \triangleright \mathbf{F}(A \wedge C)$
$\sigma \not\models B \vee C$	$\sigma \not\triangleright \mathbf{T}(B \vee C)$

we apply the rule related to $\mathbf{T}(B \vee C)$

$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}A, \mathbf{T}B\}$

$\sigma = \{A \rightarrow \mathit{true}, B \rightarrow \mathit{true}, \ldots\}$

$\sigma \models A \wedge B \;|\; \sigma \not\triangleright \mathbf{F}(A \wedge B)$

the set is contradictory.

$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}A, \mathbf{T}C\}$

$\sigma = \{A \rightarrow \mathit{true}, B \rightarrow \mathit{true}, \ldots\}$

$\sigma \models A \wedge C \;|\; \sigma \not\triangleright \mathbf{F}(A \wedge C)$

the set is contradictory.

swffs whose intuitionistic truth coincides with classical truth

Example

$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}(B \vee C), \mathbf{T}A\}$

$\sigma = \{A \rightarrow true, B \rightarrow false, C \rightarrow false\}$

$\sigma \not\models A \wedge B$	$\sigma \rhd \mathbf{F}(A \wedge B)$
$\sigma \not\models A \wedge C$	$\sigma \rhd \mathbf{F}(A \wedge C)$
$\sigma \not\models B \vee C$	$\sigma \not\rhd \mathbf{T}(B \vee C)$

we apply the rule related to $\mathbf{T}(B \vee C)$

$S = \{\mathbf{F}(A \wedge B), \mathbf{F}(A \wedge C), \mathbf{T}A, \mathbf{T}B\}$

$\sigma = \{A \rightarrow true, B \rightarrow true, \ldots\}$

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$\sigma \models A \wedge C \mid \sigma \not\rhd \mathbf{F}(A \wedge C)$

the set is contradictory.

Permutations 1

Example

$S = \{\mathbf{F}(A \to B), \mathbf{F}(C \to D)\}$

$\langle P, \leq, \rho, \Vdash \rangle$, $P = \{\rho\}$, $\Vdash = \emptyset$.

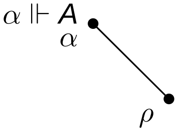
Permutations 1

Example

$S = \{\mathbf{F}(A \to B), \mathbf{F}(C \to D)\}$
$\langle P, \leq, \rho, \Vdash \rangle$, $P = \{\rho\}$, $\Vdash = \emptyset$.

$S_1 = \{\mathbf{T}A, \mathbf{F}B\}$
$\langle P, \leq, \rho, \Vdash \rangle$,

- $P = \{\rho, \alpha\}$,
- $\Vdash = \{\alpha \Vdash A\}$



$S_2 = \{\mathbf{T}C, \mathbf{F}D\}$
$\langle P, \leq, \rho, \Vdash \rangle$,

- $P = \{\rho, \alpha, \beta\}$,
- $\Vdash = \{\alpha \Vdash A, \beta \Vdash C\}$

Permutations 1

Example

$S = \{\mathbf{F}(A \to B), \mathbf{F}(C \to D)\}$

$\langle P, \leq, \rho, \Vdash \rangle$, $P = \{\rho\}$, $\Vdash = \emptyset$.

$S_1 = \{\mathbf{T}A, \mathbf{F}B\}$

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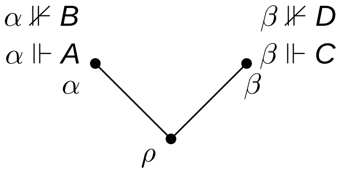

Permutations 2

Example

$\tau : \mathcal{PV} \rightarrow \mathcal{PV}$

- $\tau(C) = A$,
- $\tau(D) = B$

$\tau(S_2) = S_1$

We can avoid backtracking on S.

Permutations 3

Example

$$S = \{ \mathbf{T}(((P0 \to (P1 \vee P2)) \to (P1 \vee P2))), \mathbf{T}(((P2 \to (P1 \vee P0)) \to (P1 \vee P0))), \mathbf{T}(((P1 \to (P2 \vee P0)) \to (P2 \vee P0))), \mathbf{F}((P1 \vee (P2 \vee P0))) \}$$

$$S_1 = \{ \mathbf{T}P0, \mathbf{F}P3, \mathbf{T}(P3 \to (P1 \vee P2)), \mathbf{T}((P1 \vee P2) \to P3), \mathbf{T}(((P2 \to (P1 \vee P0)) \to (P1 \vee P0))), \mathbf{T}(((P1 \to (P2 \vee P0)) \to (P2 \vee P0))), \mathbf{F}((P1 \vee (P2 \vee P0))) \}$$

$$S_2 = \{ \mathbf{T}(((P0 \to (P1 \vee P2)) \to (P1 \vee P2))), \mathbf{T}P2, \mathbf{F}P3, \mathbf{T}(P3 \to (P0 \vee P1)), \mathbf{T}((P0 \vee P1) \to P3), \mathbf{T}(((P1 \to (P2 \vee P0)) \to (P2 \vee P0))), \mathbf{F}((P1 \vee (P2 \vee P0))) \}$$

$\tau : \mathcal{PV} \to \mathcal{PV}$

- $\tau(P0) = P2,$
- $\tau(P1) = P1$
- $\tau(P2) = P0$
- $\tau(P3) = P3$

$\tau(S_2) = S_1$

Permutations 3

Example

$$S = \{ \mathbf{T}(((P0 \to (P1 \vee P2)) \to (P1 \vee P2))), \mathbf{T}(((P2 \to (P1 \vee P0)) \to (P1 \vee P0))), \mathbf{T}(((P1 \to (P2 \vee P0)) \to (P2 \vee P0))), \mathbf{F}((P1 \vee (P2 \vee P0))) \}$$

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Permutations 3

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Permutations 3

Example

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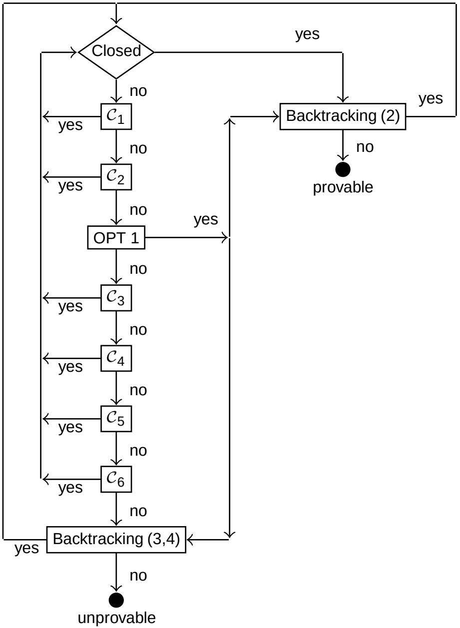
- $\tau(P3) = P3$

$\tau(S_2) = S_1$

Outline

About the implementation



Remarks

- opt2 is executed during Group 2.
- opt3 is executed during Backtracking (3,4).
- In opt3, we search for τ such that $H = \tau(H')$ and $\tau = \tau^{-1}$.

ILTP Library (T. Raths, J. Otten, C. Kreitz.)

ILTP

- Contains 274 propositional problems; time limit: 600 sec., Xeon 3.4 GHz, Mandrake 10.2.
- 128 problems are solved-Theorems.
- 109 problems are solved-Non-Theorems.
- 37 problems are unsolved.
- Five provers:
  - ft Prolog: D. Sahlin, T. Franzen, S. Haridi (Swedish Institute of Computer Science),
  - ft C: D. Sahlin, T. Franzen, S. Haridi (Swedish Institute of Computer Science),
  - LJT: R. Dyckhoff (University of St Andrews),
  - STRIP: Dominique Larchey, Daniel Mery and Didier Galmiche (LORIA),
  - PITP: A. Avellone, G. Fiorino, U. Moscato (University of Milano-Bicocca).

ILTP Library (T. Raths, J. Otten, C. Kreitz.)

ILTP

- Three domains.
- LCL (2): Logic Calculi (TPTP).
- SYN (20): Syntactic problems have no obvious semantic interpretation (TPTP).
- SYJ (252): Intuitionistic syntactic problems have no obvious semantic interpretation.
    - SYJ201 (SYJ207): de Bruijn's ,
    - SYJ202 (SYJ208): Cook pigeon-hole,
    - SYJ203 (SYJ209): Formulae requiring many contractions,
    - SYJ204 (SYJ210): Formulae with normal natural deduction proofs only of exponential size,
    - SYJ205 (SYJ211): Formulae of Korn & Kreitz,
    - SYJ206 (SYJ212): Equivalences,

Result comparison 1

	ft Prolog	ft C	LJT	STRIP	PITP
solved	188	199	175	202	215
(%)	68.6	72.6	63.9	73.7	78.5
proved	104	106	108	119	128
refuted	84	93	67	83	87
solved after:					
0-1s	173	185	166	178	190
1-10s	5	6	4	11	10
10-100s	6	7	2	11	9
100-600s	4	1	3	2	6
($>$600s)	86	75	47	43	58
errors	0	0	52	29	1

Result comparison 2

Provable

	SYJ202+1 provable	SYJ205+1 provable	SYJ206+1 provable
ft Prolog	07 (516.55)	08 (60.26)	10 (144.5)
ft C	07 (76.3)	09 (85.84)	11 (481.98)
LJT	02 (0.09)	20 (0.01)	05 (0.01)
STRIP	06 (11.28)	14 (267.39)	20 (37.64)
PITP	09 (595.79)	20 (0.01)	20 (4.07)

Refutable

	SYJ207+1 refutable	SYJ208+1 refutable	SYJ209+1 refutable	SYJ211+1 refutable	SYJ212+1 refutable
ft Prolog	07 (358.05)	08 (65.41)	10 (543.09)	04 (66.62)	20 (0.01)
ft C	07 (51.13)	17 (81.41)	10 (96.99)	04 (17.25)	20 (0.01)
LJT	03 (2.64)	08 (0.18)	10 (461.27)	08 (546.46)	07 (204.98)
STRIP	04 (9.3)	06 (0.24)	10 (132.55)	09 (97.63)	20 (36.79)
PITP	04 (11.11)	08 (83.66)	10 (280.47)	20 (526.16)	11 (528.08)

Result comparison 2

Provable

	SYJ201+1	SYJ202+1
PITP none	20 (1.29)	03 (0.01)
PITP -opt1	20 (0.03)	08 (44.59)
PITP -opt2	20 (1.67)	03 (0.01)
PITP -opt3	20 (0.03)	08 (44.21)
PITP ALL	20 (0.03)	08 (45.30)

Refutable

	SYJ207+1	SYJ208+1	SYJ209+1	SYJ211+1	SYJ212+1
PITP none	04 (43.77)	04 (2.50)	10 (596.55)	20 (526.94)	11 (527.72)
PITP -opt1	04 (44.76)	08 (93.60)	10 (325.93)	20 (558.11)	11 (548.01)
PITP -opt2	04 (12.18)	04 (2.37)	10 (311.37)	19 (293.34)	10 (88.92)
PITP -opt3	04 (11.36)	08 (94.30)	10 (591.68)	19 (291.18)	10 (92.05)
PITP ALL	04 (12.74)	08 (90.11)	10 (297.83)	19 (313.11)	10 (93.18)

Outline

Future works

- Improve permutations.
- New optimization: 234 problems are solved (85,4%).
- Implement a parallel version of the prover.