COMPUTATION: DAY 1

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Contents

1. What is computation

Wednesday, January 17, 2024.

The theory of computation is a very new subject. The Turing Machine, an important element of this theory, was defined by Alan Turing in about 1936. Its invention was a response to several important developments. However, we can look back to as far as 300 or 400 BC, to the geometric constructions, to see computation systems not tied to the artifacts of the modern world. At the time, their computer was the combination of a ruler and a compass.

We consider the the calculation of the square root of an integer n. We need to consider what an answer would look like. Today, I think probably would consider an answer to be something like,

$$\sqrt{5} = 2.23606798...$$

But that is not even an answer, just the suggestion of an approximation, that can be refined on demand. In the world of our geometric constructions, a value is the length of a segment. Therefore a solution might be: given a line segment S of length

$|S|$, in a finite sequence of application of ruler and compass actions, produce a line segment with length $\sqrt{|S|}$.

2. Ruler and Compass Constructions

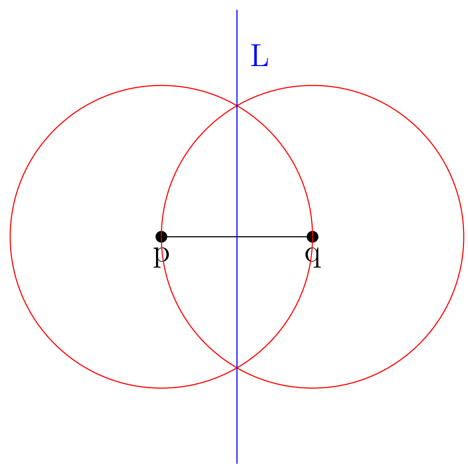
There will be only these ruler and compass actions allowed.

(1) A line can be draw by the edge of the ruler.
(2) An arc of a circle can be drawn by the compass.
(3) The ruler's edge can be aligned with one or two points.
(4) The compass can be sized to fit two points.

3. Perpendicular bisector

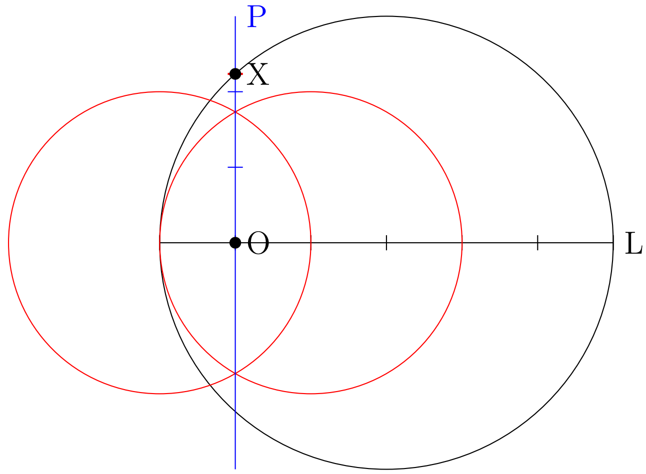
Just as we now write subroutines, which are generalized procedures to be invoked with parameters, our geometric constructions have subroutines, such as, *given two points, and a line connecting those points, construct a line perpendicular with passes through the midpoint of the two points.*.

The input is have the points drawn on the paper, p and q. The program is the steps to take with ruler and compass, and the result is a new line L drawn on the paper. Here is the subroutine's result, with the red circles being constructed for, and the blue line the result.



4. The square root of 5

We give now the procedure for constructing the length $\sqrt{5}$. Create the line L with one unit to the left of point O, and 5 units to the right of point O. Invoke the perpendicular bisector subroutine result in a perpendicular P through O. Draw a circle with diameter L. The intersection of the circle with P is X. The segment O to X has length $\sqrt{5}$.



5. Angle Bisectors and Trisectors

Given two lines L_1 and L_2 intersecting at one point X, draw a line through X that bisects the angle between L_1 and L_2. However, it is not possible to trisect the angle. The rule and compass constructions can compute square roots, angle bisectors, but not angle trisectors. Constructs we sought for almost three millennia until proved impossible by Pierre Wantzel in 1837.

Friday, 19 January 2024

6. Dr. Nim

The Dr. Nim toy was patented by John Thomas "Jack" Godfrey[1] in 1968[2], and marketed by the E.S.R. Corporation of Montclair N.J. a round 1966.[3] The company describes Dr. Nim as *a "binary digital computer" specially designed to play the game of Dr. Nim.*[4]

The game board a plastic table a little bit similar to a Pinball machine as it has channels to guide marbles and 6 plastic pieces moved by the traveling marble. It is indeed a mechanical computer fueled by the energy of a falling marble. As the marble passes three wing shaped pieces, let's call them a, b and c, they flip in interlocking

[1] `https://www.legacy.com/us/obituaries/lsj/name/john-godfrey-obituary?id=23927195`

[2] Patent 3390471A, 1968

[3] *E.S.R., Inc was founded several years ago by three scientists and engineers who wanted to provide toys and educational devices that would have those qualities that lead children and adults to fuller and more successful lives — and yet bring enjoyment to all.*

[4] *How to play Dr. Nim*, `https://www.cs.miami.edu/home/burt/learning/csc427.242/docs/491_Dr-Nim-Manual5b15d.pdf`, 1966.

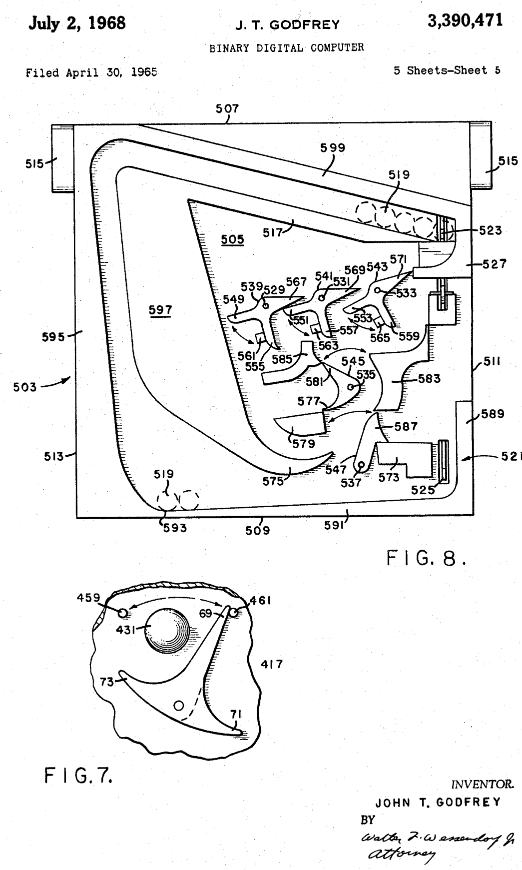


FIGURE 1. Dr. Nim US3390471, 1968

ways to cycle through 4 configurations. Let's write a capital letter when the wing is flipped up, then the wings make the following transitions with each passage of a marble,

$$a\,b\,c \longrightarrow A\,b\,c \longrightarrow a\,B\,c \longrightarrow a\,b\,C \longrightarrow a\,b\,c.$$

The other moving pieces are at locations in the channel to drive forward the narrative of the gameplay. One piece can direct a marble to launch more marbles, and when in this position it is Dr. Nim's turn. Dr. Nim will strike that piece and turn it to signal it is now the player's turn. The standard setup is to start with 15 marbles and the player and Dr. Nim take turns launching one to three marbles down the game table. The player taking the last marble losses.

The strategy for winning can be diagramed with a list of marbles painted white or black. If in this list, whether it be Dr. Nim or his human opponent, if they begin their turn with a black marble perfect game-play will end with a lose for that player. However, in the case of a white marble, that player playing correctly will go on to win.

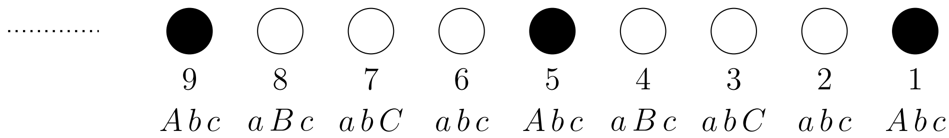


FIGURE 2. Dr. Nim strategy

Theorem 6.1. The player that begins on a black marble losses.

Proof: Between that player and his opponent, the combined number of marbles taken by both there plays can be 4. This will return the play to black. This continues until the black marble is the last marble. □

Theorem 6.2. The player that begins on a white marble wins.

Proof: That player can remove exactly the number of marbles to leave the opponent on a black marble. See the above theorem. □

The Dr. Nim game board identifies the black marble with the wing position $A\,b\,c$, and this is precisely the arrangement that triggers the change of player action. Left with any other wing position, the board channels marbles over the release lever until wing position $A\,b\,c$ is achieved.

One plastic piece, the *equalizer*, ruins Dr. Nim's strategy if it goes first, allowing the second player a chance to win. And the channeling also provides for Dr. Nim, faced with a losing situation, to only take a single marble, biding its time for a player misstep.

7. Dr. Nim and Finite Automata

As the game maker stated, the plastic Dr. Nim device is a binary digital computer. But it is a very restricted sort of computer called a *Finite State Machine*, also known as a *Finite Automata*. Formally, a finite automata is the 5-tuple

$$M = \langle\, Q, \Sigma, \delta, q_o, F\,\rangle$$

where Q is a finite set of states, Σ is a finite set called the alphabet, δ is the transition function, $\delta : Q \times \Sigma \longrightarrow Q$, a distinguished state at which the computation begins, $q_o \in Q$ and a set of states $F \subseteq Q$ such that, if the computation ends at a state in F the computation is *accepting*.

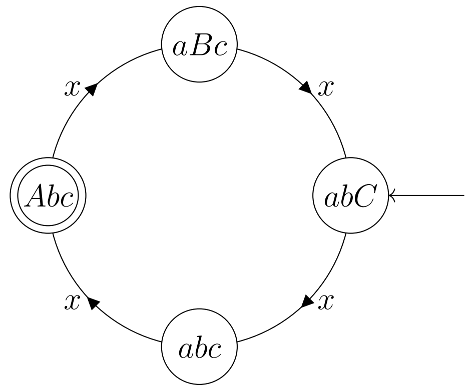


FIGURE 3. Dr. Nim as a finite automata

Definition 7.1. A computation by finite automata $M = \langle\, Q, \Sigma, \delta, q_o, F\,\rangle$, on string $s \in \Sigma^*$ is denoted $q_o \xrightarrow{\sigma} q_f$ according to the rules,

(1) $q \xrightarrow{\epsilon} q$, where ϵ is the empty string,
(2) $q \xrightarrow{\sigma} q'$, where $\sigma \in \Sigma$ and $\delta(q, \sigma) = q$;,
(3) $q \xrightarrow{s} q'$, where $s \in \Sigma^*$, and $q \xrightarrow{\sigma} q'' \xrightarrow{s'} q'$, with $s = \sigma s'$ and $\sigma \in \Sigma$.

Definition 7.2. The language of the finite automat is a subset of Σ^*, $\mathcal{L}(M) \subseteq \Sigma^*$ defined by,

$$\mathcal{L}(M) = \{\, s \in \Sigma^* \,|\, q_o \xrightarrow{s} q_f \text{ where } q_f \in F\,\}$$

A language $L \subseteq \Sigma^*$ is *regular* is there is a finite automata M such that $L = \mathcal{L}(M)$.

There is visual convention for drawing a finite automata as a labeled, directed graph. Each state is a node, and edges between states are labeled with the element of the alphabet that causes the transition between those statses. A double circled node is final a state, and a single arrow from nowhere is required to point to the start state.

Shown is a finite automata for the Dr. Nim game when the game is started with 15 marbles. We show only the four states for the four wing configurations that are encountered in normal game play. The elements for the game narrative are eliminated.

The travel of a marble is the action labelled x. When turned over to Dr. Nim (neglecting the action of the equalizer) the game will make transitions until it is in the final state. The game is then turned over to the player in a losing game. The language of this game, then, is the any string of x's that go from the start state to

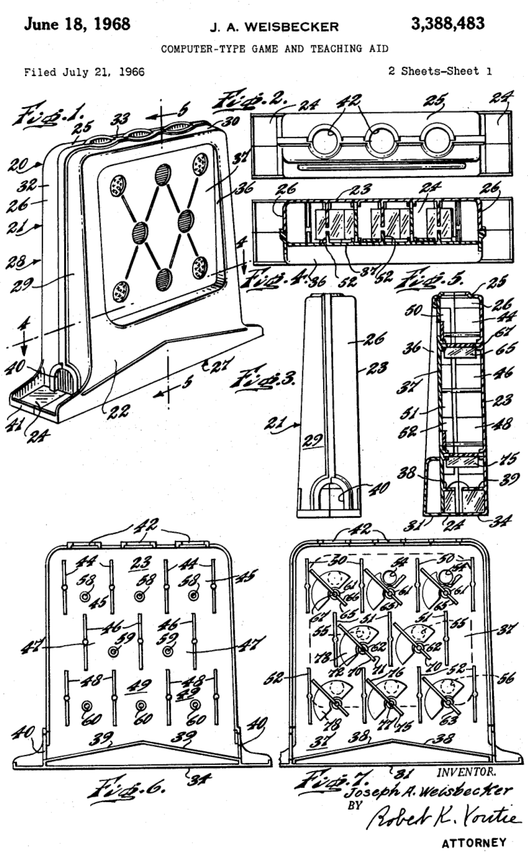


FIGURE 4. Think-A-Dot US566881A filed 1966

the final state. Define the free multiplication of symbol x as concatenation,

$$x^i \equiv \overbrace{x\,x\,\ldots\,x}^{i}.$$

then this language is,

$$\text{Nim} = \{\, x^i \mid i = 4k + 2 \text{ for a non-negative integer } k \,\}$$

8. Think-A-Dot

Monday, 22 January 2024

The Think-A-Dot was created by Joseph Weisbecker (patent US566881A,1966) and marketed by E.S.R. Inc, as a *Computer Action game*,

> *This remarkable color pattern games uses a plastic digital computer that changes colors of its spots (Flip-Flops) by dropping marbles into the three hose on the top.*[5]

The eight spots are arranged in rows 3, 2, 3, and the traveling marble changes the color of every spot is goes past. Also, the color of the spot directs the marble left or right.

The game is starting from some configuration to choose the order of ball drops to arrive at a stated final configuration: for instance, to make all the windows show yellow, or all the windows show blue. For the analysis I wish to thank Jaap Scherphuis for his webpage. [6]

If two marbles are dropped into a certain hole, the spot under that marble will change twice, returning to its original color. If four marbles are dropped into a certain hole, of course the spot under that marble will finish with its original color, but also will the spots in the second row. What ever happens in the second row with the first two marbles happens again with the second two marbles, brining the colors back to the original. And finally with a sequence of eight marbles in the same hole, all colors return to their original.

This gives a strategy. Drop one marble in each of the holes whose top color you wish to change. Then proceed with two marbles in the hole to invert the second row colors, leaving the first row unchanged. Then proceed with four marbles in a hole to invert the last row colors, leaving the other rows unchanged.

9. Exercises

(1) In the geometric construction of the square root, generalize from $\sqrt{5}$ to $\sqrt{n}$ for arbitrary positive integers n.
(2) Prove the geometric construction of the square root.
(3) Given an angle bisector construction.
(4) The Think-A-Dot has the property that if a marble is repeatedly dropped into the same hole, the pattern repeats at exactly every 8th marble. Prove this.
(5) Write out the transition function δ for the Nim finite automata.

[5] Think a Dot flyer in box.
[6] Jaap Scherphuis `https://www.jaapsch.net/puzzles/thinkadot.htm`